9.7.15] Find the Maclaurin series for the function

$$I(x) = \int_0^x \frac{\sin t}{t}\, dt$$

Solution:

$$\frac{\sin t}{t} = \frac{1}{t} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} t^{2n+1}, \quad \text{valid for } t \neq 0,$$

$$I(x) = \int_0^x \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} t^{2n}\, dt$$

$$= \sum_{n=0}^{\infty} \int_0^x \frac{(-1)^n}{(2n+1)!} t^{2n}\, dt$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{x^{2n+1}}{2n+1}.$$