9.3.18| Does $S=\sum_{n=1}^{\infty}\frac{n^4}{n!}$ converge or diverge?

Solution

$$\frac{(n+1)^4}{(n+1)!}\Big/\frac{n^4}{n!}=\frac{(n+1)^4}{n^4}\cdot\frac{n!}{(n+1)!}=\frac{(n+1)^3}{n^4}\xrightarrow{n\to\infty}0$$

Thus, by the ratio test: $S<\infty$