

9.3.11) Does  $S = \sum_{n=1}^{\infty} \frac{1}{\pi^n - n^{\pi}}$  converge or diverge?

Solution

$$\frac{1}{\pi^n - n^{\pi}} = \frac{1}{\pi^n \left(1 - \frac{n^{\pi}}{\pi^n}\right)} \sim \frac{1}{\pi^n}$$

$$S \approx \sum \frac{1}{\pi^n} < \infty$$

Limit Comparison Test

$$\lim \frac{\frac{1}{\pi^n - n^{\pi}}}{\frac{1}{\pi^n}} = \lim \frac{1}{1 - \frac{n^{\pi}}{\pi^n}} = 1 < \infty.$$

Thus  $S < \infty$ .