7.9.19] $x^2y' + y = x^2e^{1/x}$, $y(1) = 3e$

SOLUTION

$y' + \frac{1}{x^2}y = e^{1/x} \quad | \cdot e^{\int \frac{dx}{x^2}} = e^{-\frac{1}{x}}$

$e^{-1/x}y' + e^{-1/x}\frac{1}{x^2}y = 1$

$(ye^{-1/x})' = 1$

$ye^{-1/x} = x + C$

$y = (x+C)e^{1/x}$

In. cond: $y(1) = 3e \Rightarrow 3e = (1+C)e \Rightarrow C = 2$

Thus $y(x) = (x+2)e^{1/x}$.