

3.1:34 Anta f og g har inverse f^{-1} og g^{-1} .
Vis at kompositten $f \circ g$ har inverse

$$(f \circ g)^{-1} = g^{-1} \circ f^{-1}$$

Løsning: Viser at $f \circ g$ er 1-1:

$$\text{Anta at } (f \circ g)(x_1) = (f \circ g)(x_2) \quad x_1, x_2 \in D(f \circ g)$$

$$\Rightarrow f(g(x_1)) = f(g(x_2))$$

$$\Rightarrow g(x_1) = g(x_2) \quad \text{fordi } f \text{ er 1-1}$$

$$\Rightarrow x_1 = x_2 \quad \text{fordi } g \text{ er 1-1}$$

Finder inversen:

$$y = (f \circ g)^{-1}(x)$$

$$\Rightarrow x = (f \circ g)(y) = f(g(y))$$

$$\Rightarrow f^{-1}(x) = f^{-1}(f(g(y))) = g(y)$$

$$\Rightarrow g^{-1}(f^{-1}(x)) = g^{-1}(g(y)) = y$$

$$\Rightarrow (f \circ g)^{-1}(x) = y$$

$$= g^{-1}(f^{-1}(x))$$

$$= (g^{-1} \circ f^{-1})(x)$$

□ II