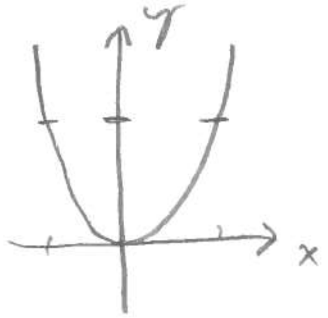
Invertere noe ikke-inverterbart (kap 3)

<u>ex</u>

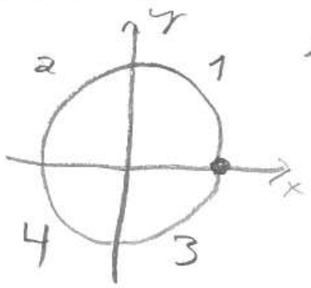
$y = x^2$ $\rightarrow$ $x = \sqrt{y}$ $\quad y \in [0, \infty),\ x \in [0, \infty)$

$x \in (-\infty, \infty)$ $\rightarrow$ $x = -\sqrt{y}$ $\quad y \in [0, \infty),\ x \in (-\infty, 0]$

$y \in [0, \infty)$



<u>ex</u>



$x^2 + y^2 = 1$

1) $y = \sqrt{1 - x^2}$ $\quad x \geq 0$

2) $y = \sqrt{1 - x^2}$ $\quad x \leq 0$

3) $y = -\sqrt{1 - x^2}$ $\quad x \geq 0$ Def Verdi

4) $y = -\sqrt{1 - x^2}$ $\quad x \leq 0$

Def. 3) $x \in [0, 1]$, Verdi 3) $y \in [-1, 0]$

Kap 3.5

$f(x) = \sin(x) \qquad x \in \mathbb{R} \qquad f(x) \in [-1, 1]$

$g(x) = \sin(x) \qquad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \qquad \mathrm{Sin}(x)$

$g^{-1}(x)$ finnes

$g^{-1}(x) = \arcsin(x) \quad = \quad \sin^{-1}(x) \neq \frac{1}{\sin(x)}$

$\sin^2(x) = (\sin(x))^2$

Oppgave

a) Finn $\sin(2\pi) = 0$

b) ~~arcsin~~ $g(2\pi)$ ikke def

c) $\sin\left(\frac{\pi}{2}\right) = 1$

d) $\arcsin(1) = \frac{\pi}{2}$

e) $\sin\left(\arcsin\left(\frac{1}{7}\right)\right) = \frac{1}{7}$

f) $\arcsin\left(\sin\left(\frac{\pi}{7}\right)\right) = \frac{\pi}{7}$

g) $\arcsin\left(\underbrace{\sin\left(\frac{68}{51}\pi\right)}_{\text{"ikke } g(x)\text{"}}\right)$

ikke nødv. $\frac{68}{51}\pi$

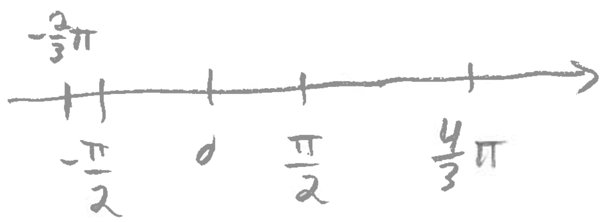
$-\frac{1}{3}\pi = -\frac{17}{51}\pi$

$\arcsin(x) \qquad x \in [-1, 1]$

$\frac{68}{51}\pi = \frac{51}{51}\pi + \frac{17}{51}\pi = \pi + \frac{17}{51}\pi = \frac{4}{3}\pi$

$= \pi + \frac{1}{3}\pi$

$\rightarrow \pi + \frac{1}{3}\pi - 2\pi = -\pi + \frac{1}{3}\pi = -\frac{2}{3}\pi$



Konsekvens for regning

Løs $\sin\left(2\theta + \frac{\pi}{2}\right) = \frac{1}{\sqrt{2}}$

$$\arcsin\left(\sin\left(2\theta + \tfrac{\pi}{2}\right)\right) = \arcsin\left(\tfrac{1}{\sqrt{2}}\right)$$

$$\underbrace{2\theta + \frac{\pi}{2} + k\cdot 2\pi}_{\in\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]} = \frac{1}{4}\pi$$

$\cancel{\in\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]}$

$k = 0, 1, 2, 3 \ldots, -1, -2, -3, \ldots$

$k \in \mathbb{Z}$

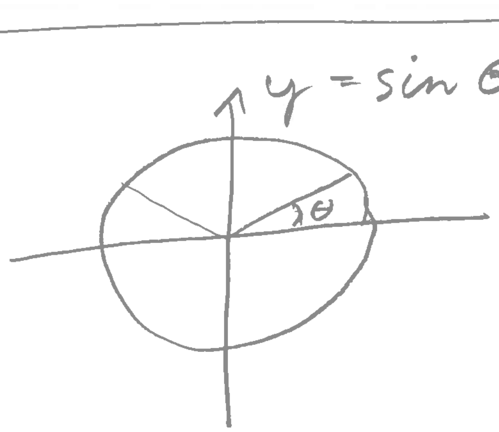
$$2\theta = \frac{1}{4}\pi - \frac{\pi}{2} - k\cdot 2\pi$$

$$\theta_k = -\frac{1}{8}\pi - k\cdot\pi$$

$$\theta_1 = -\frac{1}{8}\pi - \pi$$

$$\theta_0 = -\frac{1}{8}\pi$$

θ_{11} $\quad$ $\theta_{11} = -\frac{1}{8}\pi - 11\cdot\pi$



$$\sin\theta = \sin(\pi - \theta)$$

Derivasjon av arctan

Finn $\frac{d}{dx}\arctan(x)$

$$y = \tan x$$
$$x = \arctan y$$

løsn

$$g(y) = \arctan y = f^{-1}(y)$$

$$f(x) = \tan(x) \qquad \frac{d}{dy}\arctan y = ?$$

Implisitt

$$\frac{d}{dy}x = \frac{d}{dy}\arctan y$$

Digresjon

$$\frac{d}{dx}\tan x = \frac{d}{dx}\frac{\sin x}{\cos x}$$

$$= \frac{\cos x \cdot \cos x + \sin x \cdot \sin x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x} = 1 + \tan^2 x$$

Forsøk 2

$$\frac{dy}{dy} = \frac{d}{dy}\tan x$$

$$1 = (1 + \tan^2 x)\frac{dx}{dy}$$

$$\frac{dx}{dy} = \frac{1}{1 + \tan^2 x}$$

$$\frac{dx}{dy} = \frac{1}{1 + y^2} = \frac{d}{dy}\arctan y$$

Det vil si $\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$

3.5. EX 8

Finn $\frac{d}{dx}\tan^{-1}\left(\frac{x}{a}\right)$, og $\int \frac{1}{x^2+a^2}\,dx$

løsn

$y = \tan^{-1}\left(\frac{x}{a}\right)$ $\qquad u = \frac{x}{a}$

$y = \tan^{-1}(u)$ $\qquad u' = \frac{1}{a}$

$$\boxed{\frac{dy}{dx}} = \frac{1}{1+u^2}\cdot\frac{d}{dx}u \;\boxed{= \frac{dy}{du}\cdot\frac{du}{dx}}$$

$$= \frac{1}{1+u^2}\cdot\frac{1}{a} = \frac{1}{1+\left(\frac{x}{a}\right)^2}\cdot\frac{1}{a}$$

$$\frac{dy}{dx} = \frac{a^2}{a^2+x^2}\cdot\frac{1}{a} = \frac{a}{a^2+x^2}$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\int \frac{a}{a^2+x^2}\,dx$$

$$= \frac{1}{a}\,y = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) \quad (+C)$$

Kap 3.6

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = y$$

$$y = \tanh(x)$$

$$x = \tanh^{-1}(y) \qquad \text{Finn } \frac{dx}{dy}$$

$$\frac{dy}{dx} = \frac{e^x + e^{-x}}{e^x + e^{-x}} - 1 \cdot \frac{e^x - e^{-x}}{(e^x + e^{-x})^2} \cdot (e^x - e^{-x})$$

$$= 1 - \frac{(e^x - e^{-x})^2}{(e^x + e^{-x})^2} = 1 - \tanh^2(x)$$

$$= 1 - y^2$$

$$\frac{dx}{dy} = \frac{1}{1-y^2} = \frac{d}{dy} \operatorname{arctanh}(y)$$

e^x, $\ln x$ og ulikheter

$y = e^x \qquad x = \ln y \qquad (y > 0)$

$\frac{dy}{dx} = e^x \qquad \frac{dx}{dy} = \frac{1}{y} = \frac{1}{e^x}$

Monotone, stigende, inverterbare

$a < b \qquad a \leq b$

$e^a < e^b \qquad e^a \leq e^b$

$a < b \qquad a \leq b$

$\ln a < \ln b \qquad \ln a \leq \ln b$

Ex

$e^{x^2} > 1 \qquad \neq (e^x)^2 = e^{2x}$

$= e^{(x^2)}$

$\ln e^{x^2} > \ln 1$

$x^2 > 0$

$x \in (-\infty, 0) \cup (0, \infty)$

$x \in \mathbb{R} \setminus \{0\}$