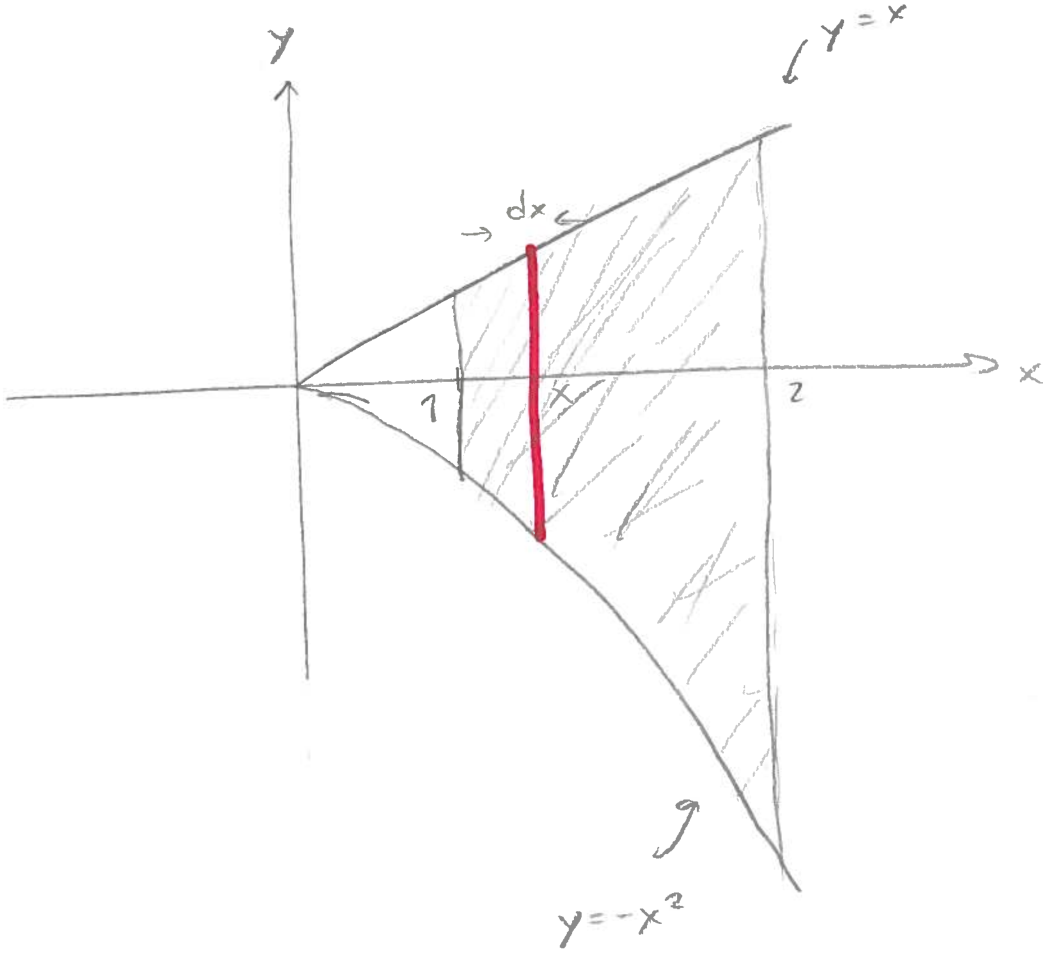
ES 22

(a)



Sylinderskallmetoden:

~~tykkelse~~: dx

høyde: $x + x^2$

omkrets: $2\pi x$

volum: $dV = 2\pi x(x+x^2)\,dx$

så

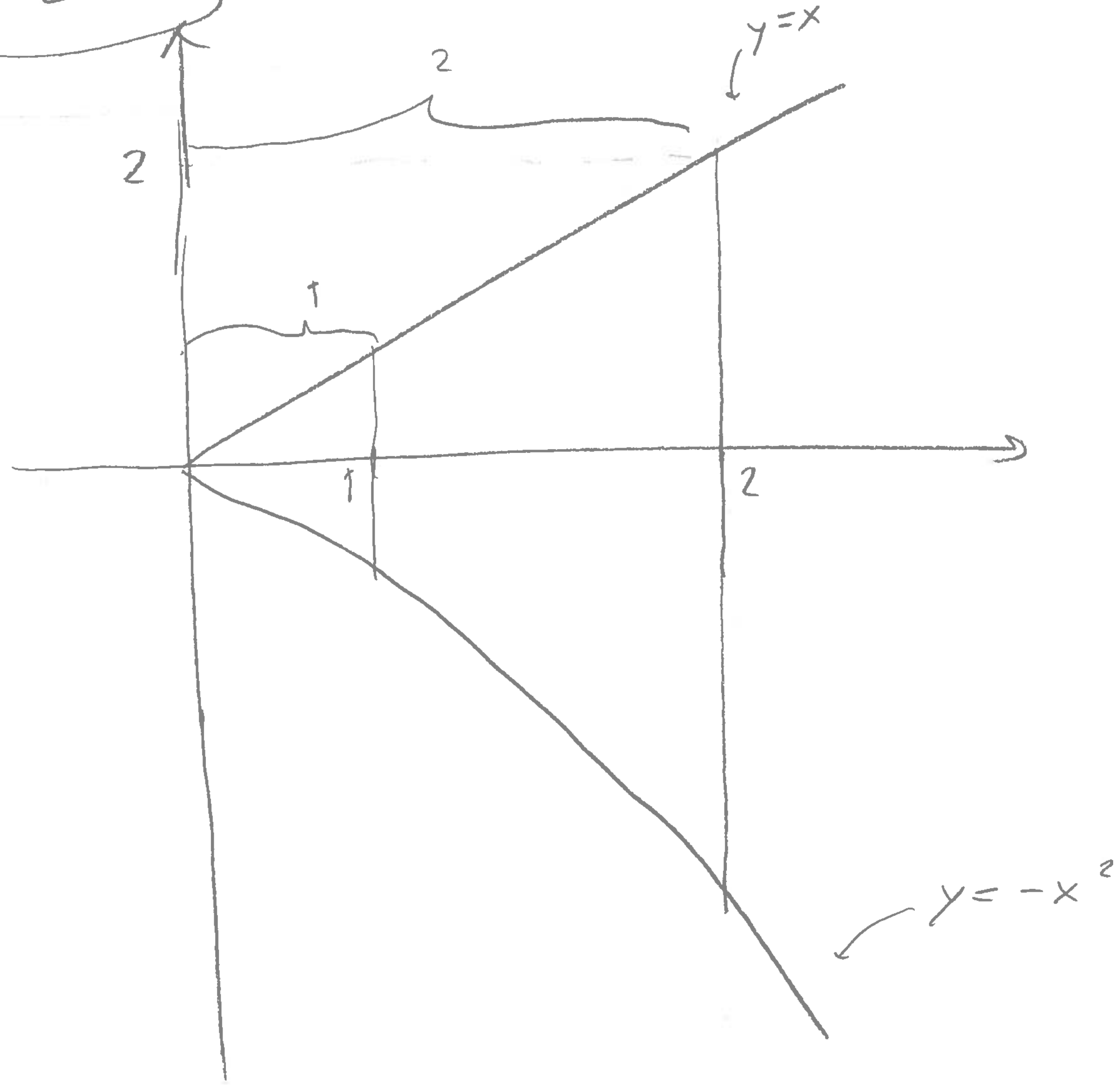
$$V = \int_{x=1}^{x=2} dV = \int_1^2 2\pi x(x+x^2)\,dx$$

$$= 2\pi \left[\frac{1}{3}x^3 + \frac{1}{4}x^4\right]_1^2 = 2\pi\left(\frac{8}{3} + 4 - \left(\frac{1}{3}+\frac{1}{4}\right)\right)$$

$$= 2\pi\left(\frac{20}{3} - \frac{7}{12}\right) = 2\pi\left(\frac{80-7}{12}\right) = \underline{\underline{\frac{73\pi}{6}}}$$

ES 22

(b)



Deler opp problemet:

Areal = (arealet som fås ved å rotere $y=x$)

+ (arealet fra $y=-x^2$)

+ (arealet av den lille sylinderen) med radius=1

+ (arealet av den store sylinderen) med radius=2

$$= \int_1^2 2\pi x \sqrt{1+\left(\frac{d}{dx}x\right)^2}\,dx + \int_1^2 2\pi x \sqrt{1+\left(\frac{d}{dx}(-x^2)\right)^2}\,dx$$

$\int 2\pi \cdot \text{radius} \cdot ds$

$$+ 2\pi\cdot 1\cdot(1+1) + 2\pi\cdot 2\,(2+4)$$

høyde høyde

$$= 2\pi \int_1^2 x \sqrt{1+1}\, dx \quad + 2\pi \int_1^2 x \sqrt{1+4x^2}\, dx$$

$$+ 4\pi + 24\pi$$

$u = 1 + 4x^2$
$du = 8x\, dx$

$$= 2\sqrt{2}\pi \cdot \frac{1}{2}x^2 \Big|_1^2 + \frac{2\pi}{8} \int_5^{17} \sqrt{u}\, du$$

$$+ 28\pi$$

$$= \sqrt{2}\,\pi\,(4-1) + \frac{\pi}{4} \cdot \frac{2}{3} u^{3/2} \Big|_5^{17} + 28\pi$$

$$= 3 \cdot \sqrt{2}\,\pi + \frac{\pi}{6}\left(17^{3/2} - 5^{3/2}\right) + 28\pi$$

$$\approx 132.14$$