

Eksempel

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STEG 1: Sjekk grad.

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Graden til teller: 2

Graden til nevner: 4

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STEG 2: Fullstendig faktorisering av nevner

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Graden til teller: 2

OK

Graden til nevner: 4

STEG 2: Fullstendig faktorisering av nevner

$$\begin{aligned} Q(x) &= (x^2 + 2x - 3)(x^2 + 1) \\ &= (x - 1)(x + 3)(x^2 + 1) \end{aligned}$$

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STEG 3: Splitt opp i delbrøker (etter visse regler).

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$$\begin{aligned}\frac{P(x)}{Q(x)} &= \frac{x^2 + 4x + 1}{(x - 1)(x + 3)(x^2 + 1)} \\ &= \frac{A}{x - 1} + \frac{B}{x + 3} + \frac{C + Dx}{x^2 + 1}\end{aligned}$$

STEG 4: Løs ligningssett

$$\frac{x^2 + 4x + 1}{(x - 1)(x + 3)(x^2 + 1)} = \frac{A}{x - 1} + \frac{B}{x + 3} + \frac{C + Dx}{x^2 + 1}$$

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Får at

$$\begin{aligned} 1x^2 + 4x + 1 &= A(x + 3)(x^2 + 1) + B(x - 1)(x^2 + 1) + (C + Dx)(x - 1)(x + 3) \\ &= (A + B + D)x^3 + (3A - B + C + 2D)x^2 + \\ &\quad (A + B + 2C - 3D)x + (3A - B - 3C), \end{aligned}$$

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En (god) del regning gir

$$A = 3/4, \quad B = 1/20, \quad C = 2/5, \quad D = -4/5$$

Siden

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er

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STEG 5: Integrer

$$\int \frac{x^2 + 4x + 1}{(x^2 + 2x - 3)(x^2 + 1)} dx = \int \frac{3/4}{x - 1} + \frac{1/20}{x + 3} + \frac{2/5 - (4/5)x}{x^2 + 1} dx$$
$$= \frac{3}{4} \ln |x - 1| + \frac{1}{20} \ln |x + 3| + \frac{2}{5} (\tan^{-1}(x) - \ln |x^2 + 1|) + C$$