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$$(1+x)^m = \sum_{n=0}^{\infty} \binom{m}{n} x^n, \quad |x| < 1$$

der

$$\binom{m}{n} = \frac{m!}{n!(m-n)!}, \quad \text{dersom } m \geq 0.$$

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$$\binom{m}{0} = 1, \quad \binom{m}{1} = m, \quad \binom{m}{n} = \frac{m(m-1)(m-2)\cdots(m-n+1)}{n!}$$

Binomialrekken – Eksempel

$$\frac{1}{1+x} = (1-x)^{-1} = \sum_{n=0}^{\infty} \binom{-1}{n} x^n, \quad |x| < 1$$

$$= \binom{-1}{0} + \binom{-1}{1} x + \binom{-1}{2} x^2 + \cdots$$

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Dermed

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n.$$

Mye brukte rekker

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad |x| < 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad |x| < \infty$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}, \quad |x| < \infty$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}, \quad |x| < \infty$$

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n, \quad 1 < x \leq 1$$

$$(1+x)^m = \sum_{n=0}^{\infty} \binom{m}{n} x^n, \quad |x| < 1$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}, \quad |x| \leq 1$$