71. **Cost from marginal cost** The marginal cost of printing a poster when x posters have been printed is

$$\frac{dc}{dx} = \frac{1}{2\sqrt{x}}$$

dollars. Find $c(100) - c(1)$, the cost of printing posters 2–100.

72. (*Continuation of Exercise 71.*) Find $c(400) - c(100)$, the cost of printing posters 101–400.

73. Show that if k is a positive constant, then the area between the x-axis and one arch of the curve $y = \sin kx$ is $2/k$.

74. Find

$$\lim_{x\to 0} \frac{1}{x^3}\int_0^x \frac{t^2}{t^4+1}\,dt.$$

75. Suppose $\int_1^x f(t)\,dt = x^2 - 2x + 1$. Find $f(x)$.

76. Find $f(4)$ if $\int_0^x f(t)\,dt = x\cos \pi x$.

77. Find the linearization of

$$f(x) = 2 - \int_2^{x+1} \frac{9}{1+t}\,dt$$

at $x = 1$.

78. Find the linearization of

$$g(x) = 3 + \int_1^{x^2} \sec(t-1)\,dt$$

at $x = -1$.

79. Suppose that f has a positive derivative for all values of x and that $f(1) = 0$. Which of the following statements must be true of the function

$$g(x) = \int_0^x f(t)\,dt?$$

Give reasons for your answers.

a. g is a differentiable function of x.

b. g is a continuous function of x.

c. The graph of g has a horizontal tangent at $x = 1$.

d. g has a local maximum at $x = 1$.

e. g has a local minimum at $x = 1$.

f. The graph of g has an inflection point at $x = 1$.

g. The graph of dg/dx crosses the x-axis at $x = 1$.

80. **Another proof of The Evaluation Theorem**

a. Let $a = x_0 < x_1 < x_2 \cdots < x_n = b$ be any partition of $[a, b]$, and let F be any antiderivative of f. Show that

$$F(b) - F(a) = \sum_{i=1}^{n} [F(x_i) - F(x_{i-1})].$$

b. Apply the Mean Value Theorem to each term to show $F(x_i) - F(x_{i-1}) = f(c_i)(x_i - x_{i-1})$ for some c_i in the interval (x_{i-1}, x_i). Then show that $F(b) - F(a)$ is a Riemann sum for f on $[a, b]$.

c. From part b and the definition of the definite integral, show that

$$F(b) - F(a) = \int_a^b f(x)\,dx.$$

COMPUTER EXPLORATIONS

In Exercises 81–84, let $F(x) = \int_a^x f(t)\,dt$ for the specified function f and interval $[a, b]$. Use a CAS to perform the following steps and answer the questions posed.

a. Plot the functions f and F together over $[a, b]$.

b. Solve the equation $F'(x) = 0$. What can you see to be true about the graphs of f and F at points where $F'(x) = 0$? Is your observation borne out by Part 1 of the Fundamental Theorem coupled with information provided by the first derivative? Explain your answer.

c. Over what intervals (approximately) is the function F increasing and decreasing? What is true about f over those intervals?

d. Calculate the derivative f' and plot it together with F. What can you see to be true about the graph of F at points where $f'(x) = 0$? Is your observation borne out by Part 1 of the Fundamental Theorem? Explain your answer.

81. $f(x) = x^3 - 4x^2 + 3x, \quad [0, 4]$

82. $f(x) = 2x^4 - 17x^3 + 46x^2 - 43x + 12, \quad \left[0, \frac{9}{2}\right]$

83. $f(x) = \sin 2x \cos \frac{x}{3}, \quad [0, 2\pi]$

84. $f(x) = x\cos \pi x, \quad [0, 2\pi]$

In Exercises 85–88, let $F(x) = \int_a^{u(x)} f(t)\,dt$ for the specified a, u, and f. Use a CAS to perform the following steps and answer the questions posed.

a. Find the domain of F.

b. Calculate $F'(x)$ and determine its zeros. For what points in its domain is F increasing? Decreasing?

c. Calculate $F''(x)$ and determine its zero. Identify the local extrema and the points of inflection of F.

d. Using the information from parts (a)–(c), draw a rough hand-sketch of $y = F(x)$ over its domain. Then graph $F(x)$ on your CAS to support your sketch.

85. $a = 1, \quad u(x) = x^2, \quad f(x) = \sqrt{1 - x^2}$

86. $a = 0, \quad u(x) = x^2, \quad f(x) = \sqrt{1 - x^2}$

87. $a = 0, \quad u(x) = 1 - x, \quad f(x) = x^2 - 2x - 3$

88. $a = 0, \quad u(x) = 1 - x^2, \quad f(x) = x^2 - 2x - 3$

In Exercises 89 and 90, assume that f is continuous and $u(x)$ is twice-differentiable.

89. Calculate $\frac{d}{dx}\int_a^{u(x)} f(t)\,dt$ and check your answer using a CAS.

90. Calculate $\frac{d^2}{dx^2}\int_a^{u(x)} f(t)\,dt$ and check your answer using a CAS.