



Norges teknisk-naturvitenskapelige
universitet
Institutt for matematiske fag

Fasit frivillige oppgaver
uke 39 for
TMA4100

Uke 39, 2008.

1 a) $x_{n+1} = x_n - \frac{x_n^3 - 2}{3x_n^2} = \frac{2}{3} \left(x_n + \frac{1}{x_n^2} \right)$

$x_1 \approx 1.2963$

b) $x_{n+1} = x_n - \frac{x_n - \cos x_n}{1 + \sin x_n}, \quad (f(x) = x - \cos x)$

$x_1 \approx 0.7552$

2 $a = 400$ m, $r = 400/\pi \approx 127.3$ m.

$(A = 2ar = a(1600 - 2a)/\pi)$

3 $x_1 = 0, x_2 = \ln 2. \quad (e^x = u \text{ gir } u^2 - 3u + 2 = 0)$

4 a) $f'(x) = \frac{1}{x^2 - 1} \quad (f(x) = \frac{1}{2}[\ln(x - 1) - \ln(x + 1)])$

b) $f'(x) = \frac{1}{2\sqrt{x(1-x)}} \quad ((\arcsin x)' = \frac{1}{\sqrt{1-x^2}})$

c) $f'(x) = \frac{-1}{x^2 + 1} \quad ((\arctan x)' = \frac{1}{1+x^2})$

5 a) $L = \frac{1}{2}$

b) $L = \lim_{x \rightarrow \infty} \sqrt{\frac{x^2 + 2}{(2x + 1)^2}} = \frac{1}{2}$

c) $L = \lim_{x \rightarrow 1} \frac{x - 1 - \ln x}{(x - 1) \ln x} = \frac{1}{2}$

d) $L = \frac{1}{\sqrt{e}}, \quad (\ln[(\cos x)^{1/x^2}] = \frac{\ln \cos x}{x^2})$

6 a) $\frac{x^3}{3} - \frac{1}{x} + C, \quad (\frac{x^4 + 1}{x^2} = x^2 + \frac{1}{x^2})$

b) $\cos \frac{1}{x} + C, \quad (u = \frac{1}{x}, du = -\frac{1}{x^2} dx)$