The generalized-α scheme for the evaluation of the energy dissipation in the elastic pendulum

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The generalized-α method is a time integration numeric method derived from the implicit Newmark family. Firstly elaborated in the work of Chung and Hulbert [1], this method is widely used in solving structural dynamics problems for its numerical damping property, which the user can edit. It allows to damp unwanted high frequencies and it keeps almost intact the low frequencies.
This property makes the algorithm a perfect solution to study our test case. Indeed, in the elastic pendulum we observe a low frequency in the oscillations of the pendulum itself, and a high frequency in the displacement of the elastic rod from the rigid pendulum. Using the generalized-α method, we simulate the phenomenon of dissipation without adding the physical term in the model. Hence, it allows us to have an easy physical model while the solution of the problem matches the physical behaviour.
Moreover, we also want to investigate the differences between a method with numerical damping, such as the generalized-α, and a structure preserving method. A first empirical attempt of studying such differences is done solving the elastic pendulum problem with the implicit midpoint rule. The method has second order of accuracy, as the generalized-α, but it is also structure preserving. Theoretical details on structure preserving algorithms, including preservation and dissipation of energy, can be found in the book from Hairer, Lubich and Wanner, [2].
In the present contribution, we apply the two numerical methods in the resolution of the nonlinear dynamical system describing the elastic pendulum. We compare the results with and without the physical damping to have a global overview of the problem. The study can be further extended analysing, from a theoretical point of view, the behaviour of the structure preserving integrator in presence of a damping term in the system.

References

[1] Chung, J., Hulbert, G.: A time integration algorithm for structural dynamics with improved numerical dissipation: the generalized-α method (1993)

[2] Hairer, E., Lubich, C., Wanner, G.: Geometric numerical integration: structure-preserving algorithms for ordinary differential equations, vol. 31. Springer Science & Business Media (2006)