MANLAB: A numerically-efficient continuation software for the periodic solution of nonlinear systems

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Abstract

The world of mathematics provides us with a multitude of techniques for solving any given problem; present and future research continues to develop new ones. Whether we wish to find an exact, analytical solution or a numerical approximation, there are many paths that we can take to arrive at the conclusion we seek. Because of this, when developing or sharing a mathematical technique, we must go beyond the simple "how it works" and push the "why it is better" - in short, we must learn to be mathematical salespeople.

To this end, we here present another technique for the periodic solution of nonlinear systems. This technique is described in [1] as a combination of the harmonic balance method (HBM), based on a frequency-domain formulation, and the asymptotic numerical method (ANM), which is a variation of numerical continuation. The end goal of the method is to trace the behavior of the system as efficiently as possible without sacrificing some of the phenomena that may appear (such as bifurcation points or multiple branches of the solution). A continuation technique is used in this case as it is able to detect and manage bifurcation points in the periodic solution and computes the corresponding path branches. The core concept to be introduced in this presentation is the idea of the *quadratic recast*. The quadratic recast is the idea that nonlinear or otherwise "difficult" terms such as fractions, fractional or negative exponents and roots can be reorganized in the original system of equations through the introduction of additional variables, called "auxiliary variables." Following the recast, the HBM and ANM can be applied consecutively, thereby reducing the nonlinear system to a large system of simple algebraic equations.

The technique has been implemented as a software program, called MANLAB. MANLAB is a path-following and interactive software that automates all of the steps just described; this means that the user has very little work to do in order to compute the solution. As expressed in [1], the quadratic recast is therefore the most complex step that the user must do themselves. Once the recast system is entered into the software, the user simply runs the code and is able to see the step-by-step evolution of the periodic simulation. In addition, there are many advantages of this technique, mostly relating to its significant decrease in computational cost compared to certain time-domain formulation techniques. The software (written in MATLAB) is free to download from the MANLAB website: `http://manlab.lma.cnrs-mrs.fr/spip/`. The website also contains extensive documentation, references and tutorials related to the software and the mathematical formulation behind it.

The outline of the presentation is as follows: (1) a short introduction to the software and its principal developers; (2) an introduction of the problem to be solved and select methods of solving; (3) a comprehensive outline of the HBM/ANM technique implemented in MANLAB; (4) a demonstration of the software; and (5) concluding remarks on the particular benefits of this technique.

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