Variational formulations and simulations of the 1D wave equation and the geometrically exact beam

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THREAD
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Variational integrators for mechanical systems are derived by discretizing Hamilton's principle of stationary action. The variational derivation of the integrators guarantees some favourable properties such as good energy behaviour meaning the energy error does not increase or decrease over simulation time but stays bounded. Moreover, if there are symmetries in the mechanical system (Noether's theorem), one can prove that variational integrators conserve the corresponding momentum maps. This work aims to give an introduction to Lagrangian mechanics in both continuous and discrete settings and construct variational integrators for a dynamic system, i.e. the 1-dimensional wave equation. The trapezoidal quadrature rule is used to approximate the integral of the Lagrangian function in both time and space. Next step of this work is to focus on the geometrically exact beam, whose equations can be seen as wave equations under a mathematical point of view. At the end, a discrete Lie group formulation of geometrically exact beam dynamics and its conservation properties are shown.

Literatur

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[2] François Demoures, François Gay-Balmaz, Sigrid Leyendecker, Sina Ober-Blöbaum, Tudor S. Ratiu and Yves Weinand. Discrete variational Lie group formulation of geometrically exact beam dynamics. Numerische Mathematik, 130(1), 73-123, 2015