Geometric Integration and Lie group methods, THREAD Network-wide training event 6

February 3, 2021

THIRD ASSIGNMENT. Building on assignment two, we want now to implement two more discretizations of the KdV equation and compare them with the energy-preserving method. The space discretization is the same as in assignment two, but we here want to implement the midpoint integration in time and the Kahan method.

The considered space discretization of the PDE problem gives a system of ODEs in the form

$$\mathbf{u}_t = S\nabla \tilde{H}_d(\mathbf{u})$$

where S is a skew symmetric matrix which does not depend on $\mathbf{u} \in \mathbb{R}^K$ and $\tilde{H}_d$ is a (discrete) Hamiltonian function. Suppose K is even, a usual central difference discreization of $\frac{\partial}{\partial x}$ will lead to S being nonsingular. The semi-discrete system is then a Hamiltonian system and the midpoint method is an appropriate choice for this system of ODEs since it is symplectic. The ODE system is also a quadratic polynomial in the components of $\mathbf{u}$ and therefore the Kahan discretization is a suitable choice because it is linearly implicit, and it preserves a modified energy function as well as a modified volume form.

Exercise. Provide a fair comparison of the three structure preserving discretization methods for the KdV equation. Compare their performance on test problems, check the presevation of invariants, the ability of maintaining the shape of the soliton and to reproduce the correct speed at which the soliton moves. Discuss advantages and disadvantages of the three methods.

Send your answers by e-mail to elena.celledoni@ntnu.no.