STUDENT PROJECTS IN HARMONIC ANALYSIS

I can offer a number of projects on mathematical problems which are motivated by needs of signal analysis. Currently these problems are in the scope of my research interests, so the level of difficulty can vary substantially: from some survey projects, to ones containing "real" interesting problems. It is also possible to have projects related to numerical simulation.

I maintain collaboration with a number of groups on signal analysis in Europe and USA, so successful projects may yield further international cooperation.

Here is **the list of topics** in which I can offer projects

- *Sampling and reconstruction of bandlimited signals.* This is a classical area of research, still a lot of important problems remain open. They concern signals with lacunar spectra, multidimensional signals and also signals with nonuniform behavior in the time domain.
- *Quantization of signals.* This is a respectively new area related to development of digital techniques. The way to store and/or transmit a signal is to quantize it, i.e. transform into a sequence of bits, and then (when need be) reconstruct this sequence into a signal which is close to the original one. Problems of economical and robust quantizing are of great importance, on the other hand they are related to various basic mathematical disciplines, from Fourier Analysis to Abstract Algebra.
- *Uncertainty principle*

  Uncertainty principle says that a function and its Fourier transform cannot be simultaneously "small". The origin of such type of statements lies in quantum mechanics: for a quantum particle it is impossible to determine its position and momentum simultaneously. The mathematical form of this statement is the famous Heisenberg inequality:

  $$\int_{-\infty}^{\infty} x^2|f(x)|^2dx \int_{-\infty}^{\infty} \xi^2|\hat{f}(\xi)|^2d\xi \geq 4\pi,$$

  if

  $$\int_{-\infty}^{\infty} |f(x)|^2dx = 1.$$

  There are many other versions of the uncertainity principle, see e.g.

Karlheinz Grochenig, Uncertainty principles for time-frequency representations. *Advances in Gabor analysis*, 11–30, Appl. Numer. Harmon. Anal., Birkhuser Boston, Boston, MA, 2003,
and references therein.

I offer two master projects on this topic:

(1) *Review project:* Make a survey of various known forms of the uncertainty principle and compare them.
(2) *Training project:* Obtain new form of uncertainty principle, in which localization of function on a segment $[-\pi, \pi]$, say, will be compared with localization of its Fourier coefficients.
    Comment: I have not seen this kind of result in literature, though the standard techniques should work

These projects may serve as introduction to more advanced problems.