Hierarchical Bayesian lithology/fluid prediction: A North Sea case study

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ABSTRACT
Seismic 3D amplitude variation with offset (AVO) data from the Alvheim field in the North Sea are inverted into lithology/fluid classes, elastic properties, and porosity. Lithology/fluid maps over hydrocarbon prospects provide more reliable estimates of gas/oil volumes and improve the decision concerning further reservoir assessments. The Alvheim field is of turbidite origin with complex sand-lobe geometry and appears without clear fluid contacts across the field. The inversion is phrased in a Bayesian setting. The likelihood model contains a convolutional, linearized seismic model and a rock-physics model that capture vertical trends due to increased sand compaction and possible cementation. The likelihood model contains several global model parameters that are considered to be stochastic to adapt the model to the field under study and to include model uncertainty in the uncertainty assessments. The prior model on the lithology/fluid classes is a Markov random field that captures local vertical/horizontal continuity and vertical sorting of fluids. The predictions based on the posterior model are validated by observations in five wells used as blind tests. Hydrocarbon volumes with reliable gas/oil distributions are predicted. The spatial coupling provided by the prior model is crucial for reliable predictions; without the coupling, hydrocarbon volumes are severely underestimated. Depth trends in the rock-physics likelihood model improve the gas versus oil predictions. The porosity predictions reproduce contrasts observed in the wells, and mean square error is reduced by one-third compared to Gauss-linear predictions.

INTRODUCTION
Seismic inversion of elastic rock properties has, during the last two decades, become an integral part of the exploration workflow in the oil industry as a complementary technique to conventional seismic interpretation. Transferring seismic data to lithology/fluid properties is an inverse problem with nonunique solutions, and it is complicated by observing contrasts only, observation errors, imprecise processing, and simplified forward modeling. Thus, the uncertainty related to lithology/fluid prediction is usually large, and the Bayesian framework is a natural choice, e.g., Tarantola (1987, 2005), Buland and Omre (2003a), Eidsvik et al. (2004), and Gunning and Glinsky (2007).

Houck (2002) shows that it is important to consider seismic and rock-physics uncertainties during lithology and fluid prediction from seismic data. The first uncertainty component is in the relationship between seismic data and elastic parameters (Buland and Omre, 2003a). The second component is uncertainty in the relationship between elastic parameters and reservoir properties, such as lithology/fluids (Mukerji et al., 2001; Avseth et al., 2005; Grana and Rossa, 2010).

In this article, we present a Bayesian method to invert seismic data into reservoir parameters. Stochastic inversion of seismic data into reservoir parameters such as porosity, saturation, and/or clay content using a lithology/fluid concept is considered in many studies. In an early work, Doyen (1988) estimates porosity from seismic data by using cokriging, whereas Mukerji et al. (2001) show how statistical rock-physics techniques combined with seismic information can be used to classify reservoir lithologies and pore fluids. In Eidsvik et al. (2004), a horizontal Markov random field...
prior model is included to impose spatial continuity, and Larsen et al. (2006) include a prior Markov-chain model for vertical fluid sorting when predicting lithology/fluids. Stochastic rock-physics models in Bachrach (2006) are used for locationwise joint estimation of porosity and saturation, and González et al. (2008) use a discrete lithology model based on a multipoint approach to predict lithology/fluids. A spatial model for the elastic properties is defined in Buland et al. (2008), but they do not use any spatial couplings in the lithology/fluid model. Bosch et al. (2009) estimate porosity and saturation from seismic data in a spatial setting, but no lithology/fluid model is involved. Grana and Rossa (2010) include lithology/fluid classes but do not use any spatial continuity model for the lithology/fluid classes. Ulvmoen and Omre (2010) predict lithology/fluids by using a Markov random field for spatial continuity and fluid sorting. In Rimstand and Omre (2010), the model in Ulvmoen and Omre (2010) is generalized to include a depth-dependent rock-physics model and inference of model parameters. Han et al. (2010) use a spatial lithology/fluid clustering method to predict lithology/fluids, and Sams and Saussus (2010) compare uncertainty estimates based on geostatistical and deterministic lithology inversion. For a review of work on seismic inversion for reservoir properties, we recommend Bosch et al. (2010).

Lithology/fluid prediction is usually done locationwise; thus, the presence of hydrocarbons at one location is predicted without using neighboring information, although hydrocarbon accumulation occurs in continuous pockets. Spatial continuity can be obtained by smoothing data or smoothing predictions; but when working with discrete classes, the model is highly nonlinear and smoothing may cause loss of valuable information. Consequently, the lithology/fluid prediction should be done in a spatial context. Elastic properties of rocks are strongly influenced by geologic trends, and the uncertainties in the lithology/fluids prediction can be reduced if these trends are included in the inversion model (Avseth et al., 2003).

In the current study, Bayesian inversion of seismic amplitude variation with offset (AVO) data and well observations into lithology/fluid classes, elastic properties, and porosities are considered. The lithology classes are shale and sandstone; the fluid classes capture gas, oil, or brine pore filling. The forward model is based on a convolutional, linearized Zoeppritz relation (Buland and Omre, 2003a) and a depth-varying rock-physics relation (Avseth et al., 2005); and these relations contain a set of stochastic model parameters representing vertical rock-physics depth trends, wavelets, and error variances that are inferred from one well. Hence, the model is defined in a hierarchical Bayes setting which entails that model parameters are associated with uncertainty. The prior model is a stationary Markov random field (Rimstad and Omre, 2010) with fixed parameters and no trends, capturing local spatial continuity and fluid ordering, and it reproduces the class proportions in the calibration well. The focus of the study is on improved prediction of the lithology/fluid characteristics and porosities with associated uncertainties. The uncertainty of the likelihood-model parameters are accounted for in the final uncertainty assessment. We have chosen to use a relatively simple model with few parameters so as not to overfit the observations from the well, but more complex model formulations may be suitable in cases where more prior knowledge and/or multiple wells are available.

A common sequential workflow for lithology/fluid prediction is first to predict elastic properties and then to locationwise classify lithology/fluids. We solve this inversion jointly and define a forward model that relates the lithology/fluid classes directly to the seismic AVO data by integrating out the effect of the elastic properties analytically. The sample space is discrete with a prior Markov random field model, but the likelihood model contains convolution. Hence, the inversion appears as a challenging probabilistic combinatorial problem with complicated constraints. If we have a reservoir discretized into n grid cells with k possible lithology/fluid classes, the sample space consists of \( k^n \) combinations of lithology/fluids classes — in this study, approximately 10\(^5\) possible models. In addition, we have a forward model that depends on unknown stochastic model parameters which we need to assess.

This paper can be seen as an extension of Rimstad and Omre (2010) into three dimensions, which again is based on Avseth et al. (2003), who define the rock-physics depth-trend models; Ulvmoen and Omre (2010), who define the lithology/fluid profile Markov random field models with associated simulation algorithm; and Buland and Omre (2003b), who present an approach to include model parameter uncertainty into seismic inversion. Furthermore, the study demonstrates how the lithology/fluid posterior model can be used to improve prediction of elastic properties and porosity. We demonstrate our approach on data from the Alvheim field in the North Sea.

FIELD AND DATA DESCRIPTION

The Alvheim field is a turbiditic oil and gas field of Paleocene age located on the Norwegian continental shelf in the North Sea (Figure 1 and Avseth et al., 2008, 2009). The distribution of turbidite sand lobes in the area is complex, with great depositional variability in lithofacies and rock texture, ranging from massive, thick-bedded sands to more heterogeneous, thin-bedded sand-shale intervals. The reservoir sands represent the Heimdal Member of the Lista Formation and are embedded in clay-rich Lista Formation shales.
The burial history is further affecting rock properties, and the Alvheim field is buried approximately 2 km below the seafloor, corresponding to a temperature of approximately 70°C. This is around the temperature at which we expect transition from the mechanical to the chemical compaction domain, as smectite-rich shale starts to transform to illite and quartz-rich sands begin to precipitate quartz cement. These compaction processes result in significant stiffening of the rock frame; as shown by Avseth et al. (2009), unconsolidated sands and cemented sandstones are present within the reservoir. The presence of cement at grain contacts has a large effect on fluid sensitivity of the seismic signal, and it is difficult to predict the correct pore fluid from seismic data without taking into account these diagenetic changes. Fluid distribution is also quite complex in the Alvheim field, with gas and oil present. The hydrocarbon migration history may have occurred at different episodes and from different origins, causing some lobe sands to be filled with oil, others predominantly with gas. The complex distribution of lithology and fluids makes it very challenging to conduct seismic reservoir characterization in the Alvheim field, and this is the key motivation for our integrated approach.

An amplitude-preserving processing sequence has been completed prior to our analysis to ensure that the seismic data quality is suitable for quantitative analysis. We have used near-, mid-, and far-stack cubes as input to our AVO inversion. The stacked cubes were generated from prestack-time-migrated (PSTM) common-depth-point (CDP) gathers from 1996, reprocessed and normal-moveout (NMO) corrected in 2004/2005. Radon demultiplexing has been applied to the NMO-corrected CDP gathers to remove multiples and increase signal/noise ratio (S/N). Near-, mid-, and far-angle stacks were produced corresponding to average angles (arithmetic average across trace range stacked) of 12°, 22°, and 31° at the Top Heimdal reservoir level. A mild frequency-space (f-x) deconvolution has been applied, in inline and crossline directions, to all stack volumes. Finally, trace interpolation using a Fourier interpolator has been applied on all stack volumes, and spectral equalization has been applied, in inline and crossline directions, to all stack volumes. The unknown variables in Figure 2 are the lithology/fluids \( \pi \); rock-physical depth-trend parameters \( \lambda \), wavelets \( s \), seismic elastic parameter covariance matrices \( \Sigma_m \) and seismic data covariance matrix \( \Sigma_d \). These parameters are treated as global and do not vary spatially; they are denoted \( \tau = [\lambda, s, \Sigma_m, \Sigma_d] \). We have chosen a parsimonious model to avoid overfitting the observations, but spatially varying model parameters can be used within the same model concept. The current model with dependence structures is illustrated in Figure 2.

The unknown variables in Figure 2 are the lithology/fluids \( \pi \); which are of primary interest, the global parameters \( \tau \), and the seismic elastic parameters \( m \). Based on these variables, porosity \( \phi \) also can be predicted. The observations are denoted \( o \). The inversion is solved in a Bayesian framework; hence, the posterior model for the unknown variables \( p(\pi, m, \tau | o) \) is the objective, where \( p(\cdot) \) is a generic term for probability-mass function or probability-density function. By using the dependence structure in Figure 2, the posterior model can by Bayes’ theorem be written as

\[
\pi \sim \text{distribution of lithology/fluids}
\]

\[
m \sim \text{distribution of seismic elastic parameters}
\]

\[
\tau \sim \text{distribution of rock-physical depth-trend parameters}
\]

\[
o \sim \text{distribution of observations}
\]

Figure 2. Graph of stochastic model. The nodes represent stochastic variables, and the arrows represent probabilistic dependencies.
\[ p(\pi, \mathbf{m}, \tau | \mathbf{o}) = \text{const} \times p(\mathbf{o} | \mathbf{m}, \tau) p(\pi, \tau) \]
\[ = \text{const} \times p(\mathbf{d} | \mathbf{m}, \mathbf{s}, \Sigma_d) p(\mathbf{m} | \pi, \lambda, \Sigma_m) \]
\[ \times p(\mathbf{m}_p^0 | \mathbf{m}) p(\pi_p^0 | \pi) \]
\[ \times p(\pi) p(\lambda) p(s) p(\Sigma_m) p(\Sigma_d), \]
(1)

where \( p(\mathbf{d} | \mathbf{m}, \mathbf{s}, \Sigma_d) \) is the seismic likelihood model, \( p(\mathbf{m} | \pi, \lambda, \Sigma_m) \) is the rock-physics likelihood model, \( p(\mathbf{m}_p^0 | \mathbf{m}) \) and \( p(\pi_p^0 | \pi) \) are well likelihood models, \( p(\pi) \) is the lithology/fluid prior model, \( p(\lambda) \) is the rock-physics depth-trend parameter prior model, \( p(s) \) is the wavelet prior model, \( p(\Sigma_m) \) is the elastic parameter covariance matrix prior, and \( p(\Sigma_d) \) is the seismic data covariance matrix prior. The likelihood and prior models are mostly 3D versions of the ones used in Rimstad and Omre (2010); thus, they are only briefly summarized here.

**Likelihood models**

The likelihood models are probabilistic forward models and are represented by arrows in Figure 2. The rock-physics forward model \( p(\mathbf{m} | \pi, \lambda, \Sigma_m) \) links porosity/cementation/lithology/fluids and seismic elastic properties. The model is parameterized by porosity/cementation depth trends (see Figure 3), and it is parameterized similar to the model in Ramm and Bjørlykke (1994):

\[
\phi_{sh}(\lambda, t) = \phi_{sh}^0 \exp \{-\alpha_{sh}(t - t^0)\},
\]
\[
\phi_{ss}(\lambda, t) = \begin{cases} 
\phi_{ss}^0 \exp \{-\alpha_{ss}(t - t^0)\} & \text{if } t \leq t^c \\
\phi_{ss}(t^c) - \kappa_{ss}(t - t^c) & \text{if } t > t^c
\end{cases},
\]
(2)

where \( sh \) indicates shale, \( ss \) indicates sand, \( t^0 \) is a reference depth, \( \phi_{sh}^0 \) is the porosity at depth \( t^0 \), the cementation initiates at depth \( t^c \), and \( \alpha_{sh} \) and \( \kappa_{ss} \) are regression coefficients. The depth-trend parameters are denoted \( \lambda = \{\phi_{sh}^0, \phi_{ss}^0, \alpha_{sh}, \alpha_{ss}, \kappa_{ss}, t^c\} \).

The probabilistic link between porosity/cementation/lithology/fluids and seismic elastic properties \( p(\mathbf{m} | \pi, \lambda, \Sigma_m) \) is defined by a Gaussian distribution where each cell \( t_{xy} \) is conditionally independent with expectation \( h_{xy}(t, \lambda) \) and covariance matrix \( \Sigma_m \). The expectation \( h_{xy}(t, \lambda) \) is calculated from a Hertz-Mindlin model for unconsolidated sand (Avseth et al., 2005), a Hashin-Shtrikman model for shale (Holt and Fjær, 2003), and a Dvor-kin-Nur constant/contact cement model for cemented sandstone (Dvorkin and Nur, 1996). Fluid effects are calculated by Gassmann’s relations (Gassmann, 1951). We assume only homogeneous and isotropic rocks.

The seismic likelihood model \( p(\mathbf{d} | \mathbf{m}, \mathbf{s}, \Sigma_d) \) is defined to have a form similar to Buland and Omre (2003b). The probabilistic model is Gaussian; each seismic trace is conditionally independent with expectation \( \text{WADm}_y \) and covariance matrix \( \Sigma_d \), where \( W \) is a convolution matrix based on the wavelets \( s, A \), a weak contrast approximation reflection matrix (Aki and Richards, 1980), and \( D \) is a differential matrix that gives the contrasts of the elastic properties.

We assume exact observations in the calibration well: \( \pi^w_\omega = \pi_\omega \) and \( \mathbf{m}^w_\omega = \mathbf{m}_\omega \); hence, both well likelihoods are of a Dirac form. These assumptions are justified by the errors in well observations being ignorable relative to seismic errors.

**Prior models**

The prior models represent the prior knowledge about the variables and unknown model parameters. The prior models are stationary across the field and are parameterized such that we can use efficient Gibbs samplers in a Markov-chain Monte Carlo (MCMC) setting (see, for example, Gilks et al. [1996]).

The prior model for the lithology/fluid variables \( p(\pi) \) is defined as a Markov random field (Besag, 1974; Rimstad and Omre, 2010); which assigns higher probability to outcomes with continuity and fluid sorting. The Markov random field is specified by a vertical transition matrix \( P \) and lateral coupling parameters \( \beta^l \) and \( \beta^f \) (Figure 4). The transition matrix \( P \) controls the vertical lithology/fluid relations in vertical neighborhood \( \partial^v \pi_{xy} \) (Figure 4). In our case, it ensures gravitational sorting of fluids and it represents the class proportions observed in the well. The lithology continuity parameter \( \beta^l \) determines the continuity of lithology in the sedimentary direction \( \partial^s \pi_{xy} \) (Figure 4); equivalently, \( \beta^l \) determines the fluid continuity in the horizontal direction \( \partial^f \pi_{xy} \).

The prior for the rock-physics depth-trend parameters \( p(\lambda) \) follows a truncated normal distribution to ensure that we always get valid porosity values. The prior for the wavelets \( p(s) \) is Gaussian, and the priors for the covariance matrices \( p(\Sigma_m) \) and \( p(\Sigma_d) \) are inverse Wishart distributed (Buland and Omre, 2003b).

![Figure 3. Porosity-depth-trend model. Reference time-depth is \( t_0 \), and time-depth for top of sand cementation is \( t_c \).](image-url)

![Figure 4. System of axis in prior Markov random field model. A cross section displaying vertical neighborhood \( \partial^s \pi_{xy} \) and (right) sedimentary neighborhood \( \partial^f \pi_{xy} \).](image-url)
Posterior model

We are primarily interested in predicting lithology/fluids but also elastic properties and porosity; thus, the global parameters are considered to be nuisance parameters. The posterior model for lithology/fluids and elastic properties is simultaneously defined in \( p(\pi, m, \tau | o) \) (see equation 1). The seismic elastic parameters can, however, be analytically integrated out when focusing on lithology/fluid classes, i.e., we merge the rock-physics and seismic forward model into one forward model (Rimstad and Omre, 2010). The posterior distribution \( p(\pi, \tau | o) \) can be written

\[
p(\pi, \tau | o) = p(\pi | o, \tau)p(\tau | o).
\]

(3)

We assume that the observations in the calibration well traces \( o_w \) are sufficient to estimate \( \tau = [\lambda, s, \Sigma_m, \Sigma_d] \) such that \( p(\tau | o) \approx p(\tau | o_w) \). Following Rimstad and Omre (2010), \( p(\tau | o_w) \) and \( p(\pi | o, \tau) \) can be found. By using Bayes’ theorem, \( p(\tau | o_w) \) is proportional to the well-trace parts of equation 1:

\[
p(\tau | o_w) = \text{const} \times \int \sum_{\pi_w} p(d_w | m_w, s, \Sigma_d)p(m_w | \pi_w, \lambda, \Sigma_m)
\times p(m_o^w | m_w)p(\pi^o_w | \pi_w)p(\lambda)p(s)
p(\Sigma_m)p(\Sigma_d)dm_w
= \text{const} \times \int p(d_w | m_w, s, \Sigma_d)p(m_o^w | \pi^o_w, \lambda, \Sigma_m)p(\lambda)p(s)p(\Sigma_m)p(\Sigma_d).
\]

(4)

where each trace is assumed to be conditionally independent, the well likelihood models are defined to be of Dirac form, and const is a constant with respect to \( \tau \). The expression \( p(\pi | o, \tau) \) is from using Bayes’ theorem, proportional to equation 1 when seismic elastic properties are integrated out:

\[
p(\pi | o, \tau) = \frac{p(\pi, \tau | o)}{p(\tau | o)}
\]

(6)

\[
= \text{const} \times \int p(\pi, \tau, m | o)dm
= \text{const} \times p(\pi, \tau | o),
\]

(7)

where const is a constant with respect to \( \pi \) and the integral over elastic properties being analytically tractable.

The posterior model for elastic properties can be written

\[
p(m | o) = \int \sum_{\pi} p(m, \pi, \tau | o)d\tau
= \int \sum_{\pi} p(m | o, \pi, \tau)p(\pi, \tau | o)d\tau.
\]

(8)

where \( p(m | o, \pi, \tau) \) is a Gaussian distribution (Rimstad and Omre, 2010). The posterior model \( p(m | o) \) is a mixture model. Note that it is not a Gaussian mixture due to integration over global model parameters \( \tau \).

The posterior model of porosity can be assessed similarly to the elastic properties. The porosity \( \phi | o \) is predicted by posterior mean \( \mathbb{E}[^{\phi | o}] \), which is a probability weighted average of sand and shale porosities from the depth trends in Figure 3:

\[
\mathbb{E}[^{\phi | o}] = \left\{ \begin{array}{ll}
\mathbb{E}[\phi_{sxy} | o] = \int (\phi_{sxy}(\tau, t)p(\pi_{sxy} = ss | o, \tau) + \phi_{sh}(\tau, t)p(\pi_{sxy} = sh | o, \tau))p(\tau | o)d\tau; txy \in \mathcal{L}_D \end{array} \right\}
\]

(9)

where \( ss = sg \cup so \cup sb \) is sand and \( sh \) is shale.

The posterior model is assessed by an MCMC algorithm, summarized as

\[
\text{Do } s = 1, \ldots, S
\]

Simulate \( \tau^s \) from \( p(\tau | o_w) \)
Simulate \( \pi^s \) from \( p(\pi | o, \tau^s) \)
Simulate \( m^s \) from \( p(m | o, \pi^s, \tau^s) \)
Simulate \( \phi^s \) from \( p(\phi | o, \pi^s, \tau^s) \)

From \( (\pi^s, \tau^s, m^s, \phi^s); s = 1, \ldots, S \), the posterior models \( p(\pi | o), \phi | o, p(m | o), \) and \( \phi | o \) can be assessed.

The algorithm contains a loop that provides one realization of global parameters and lithology/fluid image \( (\pi^s, \tau^s) \). The latter, \( \pi^s \), is generated by an iterative McMC algorithm from \( p(\tau | o_w) \); the latter, \( \pi^s \), is generated by an iterative McMC algorithm from \( p(\pi | o, \tau^s) \), which is dependent on the former. The McMC algorithms converge as the number of iterations tends to infinity. Realizations of \( m \) and \( \phi \) can be simulated conditionally on \( \pi^s \) and \( \tau^s \). The loop is passed \( S \) times to provide multiple realizations of \((\pi, \tau, m, \phi)\) from \( p(\pi, \tau, m, \phi | o) \). For details, see Rimstad and Omre (2010).

The model presented above is termed model a, and it includes rock-physics depth trends and spatial lithology/fluid coupling. In the evaluation, two other model formulations are used for comparison. Model b is without depth trends and with spatial coupling. In this model, the rock-physics depth trends are reduced by setting \( \alpha_{sh} = \alpha_{ss} = 0 \) and \( r^s = \infty \) (see equation 2), which includes no depth dependence of porosity and no cementation. Model c has depth trends and is without spatial coupling. In this model, the lateral coupling parameters are defined to be \( \beta^l = \beta^v = 0 \) and vertical transition matrix \( P \) contain no vertical dependence, which includes a prior model without spatial coupling.

CASE STUDY

The study is done in three dimensions on a lattice with \( 76 \times 128 \times 91 = 885,248 \) cells, covering an area of about \( 17 \times 14 \) km. We have used near-, mid-, and far-stacked seismic data at angles \( (12^\circ, 22^\circ, 31^\circ) \) in each lattice node. Moreover, we have observations from six wells in the study area. The seismic far-stack data in one line through two wells are displayed in Figure 5a, and the lithology/fluid and reservoir observations in well 1 used as a calibration well are displayed in Figure 5b. Figure 5c also contains a crossplot of observed elastic material properties in well 1 for each lithology/fluid class. The lithology/fluid classes are not well separated, which make classification complicated. Well 1 and well 2 are 24/6-2 and 25/4-7 in Figure 4, respectively. The dashed rectangle in Figure 5a identifies the target volume, and the black lines identify the wells. Due to complex turbidite sand, the sedimentary system of axis is chosen to be the same as the current system of axis. The dotted dipping line represents the depth-trend reference time \( t_0 \); no dip in \( t_0 \) is assumed perpendicular to the seismic line. The Ramms-Bjørlykke porosity-depth trends that we use as constraints in this study assume continuous subsidence and laterally invariable compaction. However, the
dipping reflector in the overburden seen in Figure 5a indicates tectonic influence and uplift that is varying laterally. We correct for this dip by introducing the reference time-depth $t_0$ in equation 2. Based on this interpretation of the burial history, we find that the target level in well 1 must have been exposed to deeper burial and higher temperatures than the same stratigraphic level in well 2. This can explain the slightly more consolidated reservoir rocks in well 1 compared to well 2 (see Avseth et al., 2009).

The primary objective is to determine $\pi$, classified as {sand-gas, sand-oil, sand-brine, shale} over the target reservoir volume. In the study, seismic data and observations in one well, well 1, will be used to assess the global model parameters $p(\tau | o_w) = p(\tau | o_w, d, \pi)$. In assessing the lithology/fluid variables, only the seismic data will be conditioned on; hence, $p(\pi | o_w, \tau) = p(\pi | d, \tau)$. Observations in five wells, wells 2–6, are kept aside for blind testing to validate the model. The model-parameter values used in the study are given in Appendix A.

The McMC algorithm for model a is run for 10,000 sweeps; a convergence plot is displayed in Figure 6. The trace plots of proportions of the four lithology/fluid classes are displayed for the first 200 sweeps for four extreme initiations. The burn-in period is defined to be the first 25 sweeps. The mixing appears to be satisfactory. The convergences for the other parameters look similar; therefore, the convergence is judged to be satisfactory. Convergence for model b is comparable to convergence for model a, whereas model c converges extremely fast because there is no spatial coupling in the prior model for $\pi$. The computing requirement

![Figure 6. Convergence plot for McMC algorithm for model a. The first 200 sweeps with four extreme initial configurations. Proportions are classified as gas (red), oil (green), brine (blue), and shale (black).]
for 10,000 sweeps for our study is about one hour on a regular workstation. The algorithm scales linearly in the number of lattice cells and can easily be implemented on a parallel processor to reduce the computation time.

**Assessment of global parameters**

Consider the assessment of global model parameters $\tau$. Seismic data and observations in only well 1 are used to assess these global parameters. The observations in the other five wells are used only in a blind test to evaluate the predictive power of the model. The resulting posterior distributions for the global model parameters and the likelihood models are illustrated in Figures 7–10.

The posterior distributions of the depth-trend parameters are displayed in Figure 7. One observes that for most parameters, the posterior distributions (solid line) are fairly peaked relative to the wide prior distributions (dashed line). This confirms that there is considerable information about the rock-physics depth trends in well 1. One exception is the posterior for the exponential sand gradient parameter $\alpha_{ss}$, which is similar to the prior since few unencumbered observations of sand occur in the well. The crossplots on the off-diagonal display positive correlations between $\phi^0_{ss}$ and $\alpha_{ss}$. This is expected because a decrease in reference time value $\phi^0_{ss}$ could partly be compensated by reducing the gradient parameter $\alpha_{ss}$ (equation 2).

Figure 8 contains elastic properties and porosity observations from wells 1 and 2, the two wells in Figure 5a, with predictions based on models with and without rock-physics depth trends. Well 1, which is used to assess the depth trends, is displayed in the upper row, with depth trends to the left and without to the right. It is obvious that the trends capture important features in the observations. Well 2, displayed in the lower row, is used as a blind test. Also here, the trends in the observations are represented very reliably by the trend model, in spite of difference in reference depth for the two wells (Figure 5a).

The wavelet shapes are supplied from the data-set owner, and the wavelets are assumed known up to a multiplicative calibration parameter $c_w$ that we assess. The calibration parameter is assumed to be identical for all angles. The wavelet forms are displayed in Figure 9a, and their frequency contents are similar. The prior and posterior distribution of the calibration parameter $c_w$ is displayed in Figure 9b. The true reflectivities are given by the contrasts in elastic parameters observed in the well-log data, and the seismic amplitudes are calibrated to these reflectivities via the calibration parameter $c_w$. The posterior model of $c_w$ (solid line) is considerably more peaked than the prior (dashed line). The posterior models for covariances $\Sigma_m$ and $\Sigma_d$ are of less interest in this study; hence, they are not explicitly displayed. Figure 9c displays the real seismic signal and expected synthetic seismic signal generated from the well observations with the estimated model, i.e., based on the expected value of $c_w$. The match between the real and synthetic signals is reasonable, and we consider it as acceptable for this study.

The depth-integrated rock-physics model estimated from well 1, which also captures the effect of $\Sigma_m$, is displayed in Figure 10. The elongated shape for sand classes is a result of cementation varying with depth. Note also the poor separation between sand-gas and sand-oil. The total S/N is estimated to $\text{Var} [\text{WAD}_h(\lambda)]/\text{Var} [\text{WAD}_m + \epsilon_d] = 1.10$, and seismic S/N is estimated to $\text{Var} [\text{WAD}_m]/\text{Var} [\epsilon_d] = 2.26$. The $h_\phi(\lambda)$ and $e_m$ are the signal and noise part of the rock-physics relation; WAD and $\epsilon_d$ are the forward function and noise part of the seismic relation. The noise is defined to be 99% wavelet colored.

![Figure 7. Prior and posterior models of depth-trend parameters $\lambda$. Diagonal: posterior distributions (solid lines) estimated from McMC realizations, prior distributions (dashed lines). Off-diagonal: crossplots of McMC realizations from posterior model.](image-url)
Figure 8. Estimated depth trends and observations. Well 1, (a) with trend and (b) without trend. Well 2, (c) with trend and (d) without trend.

Figure 9. Prior and posterior models of wavelets. (a) Wavelets shape supplied from the data set owner for 12° (black), 22° (green), and 31° (red). (b) Prior (dashed line) and posterior (solid line) models of wavelet scale parameter $c_w$. (c) Real seismic signal and expected synthetic seismic signal generated from the well observations with the estimated model.
Prediction of lithology/fluid classes

Consider the lithology/fluid prediction with associated uncertainties represented by $p(\pi | o)$. Based on this posterior model, predictions can be made, realizations generated, and probabilistic statements provided. Three alternative models are used to establish $p(\pi | o)$: (a) with rock-physics depth trends and with spatial lithology/fluid coupling, (b) without rock-physics depth trends and with spatial lithology/fluid coupling, and (c) with rock-physics depth trends and without spatial lithology/fluid coupling. Conditioning is made on seismic AVO data only. Well 1 is used for model parameters inference; the five other wells, wells 2–6, are only used in blind tests for model validation purposes. The results from the study are presented in Figures 11–13.

The predictions of gas and oil are displayed in the 3D display in Figure 11, which presents 50% isoprobabilities for sand with gas and sand with oil. This entails that, inside the red or green volumes, the probabilities are above 0.5 for sand with gas or sand with oil, respectively. Figure 11a displays sand with gas and sand with oil.

![Figure 10. Depth-integrated rock-physics model estimated from well 1: sand-gas (red), sand-oil (green), sand-brine (blue), and shale (black).](image10)

![Figure 11. Posterior model for various models: 0.5 isoprobability volumes for sand-gas (red) and sand-oil (green). (a) Model with depth trends and spatial couplings, (b) model without depth trends and with spatial couplings, and (c) model with depth trends and without spatial couplings.](image11)
predictions for model a, with depth trends and spatial coupling. Note the continuity of the fluid classes and the vertical segregation and ordering of gas and oil, which are provided by the prior Markov random field. The spatial coupling improves the classification quality because gas and oil are expected to have lateral continuity and gravitational ordering. In spite of this continuity, no unique fluid contacts across the reservoir exist; all contacts appear as local due to the turbidite origin of the reservoir. In Figure 11b containing results from model b, we ignore depth trends and get different fluid classification compared to model a. Some areas that are classified as oil in model a are now classified as gas, although the total area classified as hydrocarbons is about the same. If predictions are made with model c, without spatial coupling (Figure 11c), the resolution in the classification is much poorer than for the other models. The total area that is classified as hydrocarbons is significantly reduced, and almost no oil is identified. Gas and oil classes are assigned with less confidence, and they appear scattered due to overfitting to the observation errors in the seismic data.

Figure 12. Well lithology/fluid prediction and observations for various models. (a) Model with depth trends and spatial coupling, (b) model without depth trends and with spatial coupling, and (c) model with depth trends and without spatial coupling.
Figure 12 compares the lithology/fluid predictions to the well observations. The observed and marginal maximum posterior predictions for lithology/fluids in all of the wells for the three models are displayed. Remember that only well 1 is used to estimate the global model parameters and that only seismic data are conditioned on when making predictions; hence, wells 2–6 are blind-test wells. The results based on model a with depth trends and with spatial coupling are displayed in Figure 12a. Observations and predictions are fairly similar, particularly for the hydrocarbon columns. Shale is overpredicted below the hydrocarbon zone, however. Figure 12b contains predictions based on model b, and a similar match between observations and predictions are

![Figure 12a](image1)

![Figure 12b](image2)

Figure 13. Posterior probabilities for lithology/fluid classes for the 2D profile shown in Figure 5a for (a) model a with depth trends and spatial coupling and for (b) model c with depth trends and model without spatial coupling.
observed. The exception is well 2, where no oil is predicted, and well 6, where false gas is predicted. The rock-physics trend, capturing porosity variations and cementations, being the difference between models a and b, is obviously important for the correct prediction of hydrocarbon type. In model c without spatial coupling, the hydrocarbon volume is dramatically underestimated and no oil is identified.

Posterior probabilities for the lithology/fluid classes under the full model a and under model c without spatial coupling in the cross section in Figure 5a are displayed in Figure 13. For model a, gas and oil areas are clearly identified, but it is more difficult to distinguish between sand-brine and shale. Note also the continuity of classes and the relatively sharp transitions in the fluid prediction. The horizontal continuity and gravitational ordering enforced by the prior model are causing these features. For model c, it is much harder to identify hydrocarbon volumes.

Figure 14 displays the marginal posterior probability for lithology/fluids in wells 2 and 5. These two wells display the clearest differences for the three models in Figure 12. As in Figure 13, the sand-oil classes are clearly identified by model a with depth trends and spatial coupling (Figure 14a). Note also the difficulty in separating sand-brine and shale above and below the hydrocarbon zone, which most likely is because of sand-shale heterogeneity. The sand-shale heterogeneity will be reproduced in a set of lithology/fluid realizations generated from the model. For model b without depth trends and with spatial coupling, displayed in Figure 14b, the probabilities are comparable to model a. One notable difference in well 2 is the hydrocarbon type. Model b is not able to clearly predict sand-oil, only indicating it with low probability. Model c without spatial coupling in Figure 14c also indicates the hydrocarbons, but with very low probabilities.

Based on these blind tests in wells 2–6, we conclude that both spatial coupling and a representative rock-physics model are important for reliable predictions. The rock-physics depth trends, which capture porosity variations and cementation, improve the predictions of oil versus gas. Predictions from models without spatial coupling can be highly unreliable.

**Prediction of elastic properties and porosity**

We are from now on only using model a as the model for the lithology/fluid classes. The estimated posterior means $\mathbf{E}[m | \alpha]$ and $\mathbf{E}[\phi | \alpha]$ (see equations 8 and 9) are displayed in Figure 15a. For comparison, predictions of elastic properties based on the Gauss-linear model in Buland and Omre (2003a), where lithology/fluid classes and the uncertainty of the global parameters are ignored, are presented in Figure 15b. For this Gauss-linear model, the porosity prediction must be calculated from the density $\rho$ with a mass-balance equation:

$$\phi = \frac{\rho_m - \rho}{\rho_m - \rho_f}, \quad (10)$$
Figure 15. Posterior means for elastic properties and porosity for various models. (a) Mixture model including lithology/fluid classes, (b) Gauss-linear models.

Figure 16. Elastic properties for various models. Lithology/fluid classification according to observations in wells: sand-gas (x), sand-oil (o), sand-brine (Δ), and shale (*). Elastic properties from well (black), from mixture model (red), and from Gauss-linear model (blue).
where \( \rho_m \) is the mineral density and \( \rho_f \) is the fluid density, which here always is assumed to be water; the values are given in Table A-1. Figure 15 shows that the posterior predictions, including lithology/fluid classes, give predictions with sharper contrasts and higher extremes than the Gauss-linear predictions. The latter predictions appear as more smoothed, which is not surprising for a linear model. We need to make a joint inversion of lithology/fluids and the reservoir variables to obtain sharp contrasts.

The effect of including lithology/fluid classes, i.e., using a mixture model on the elastic properties, can be seen in Figure 16, which is related to Figure 10. We want equal symbols of mixture models (red), and Gauss-linear model (blue) to overlap the well observations (black). Well 1 is used in inference of the mixture model, and it is not surprising that red are closer to black than blue is for all lithology/fluid classes. Well 2 is a blind-test well, and here also red tends to be closer to black than blue is. The mixture model has more degrees of freedom and can adapt better to the elastic characteristics. Note that the lithology/fluid clusters in the two displays are shifted because wells 1 and 2 have different rock-physics reference depths \( t_0 \).

Figure 17. Porosity prediction and observations for various models. Mixture model including lithology/fluid classes (red): posterior mean (solid) and 0.8 prediction interval (dashed). Gauss-linear model (blue): posterior mean (solid) and posterior standard deviation (dashed). Observations in well (black).

Figure 18. Work flow of hierarchical Bayesian inversion.

Table 1. Mean square error (MSE, unit \( 10^{-3} \)) and coverage for 0.8 prediction interval for mixture model including lithology/fluid classes and Gauss-linear model.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Well 1</th>
<th>Well 2</th>
<th>Well 3</th>
<th>Well 4</th>
<th>Well 5</th>
<th>Well 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSE, mixture model</td>
<td>1.45</td>
<td>0.55</td>
<td>1.77</td>
<td>1.22</td>
<td>1.06</td>
<td>2.10</td>
</tr>
<tr>
<td>MSE, linear model</td>
<td>2.08</td>
<td>1.81</td>
<td>2.36</td>
<td>2.10</td>
<td>2.69</td>
<td>1.95</td>
</tr>
<tr>
<td>Coverage, mixture model</td>
<td>0.80</td>
<td>0.94</td>
<td>0.75</td>
<td>0.75</td>
<td>0.84</td>
<td>0.62</td>
</tr>
<tr>
<td>Coverage, linear model</td>
<td>0.88</td>
<td>0.79</td>
<td>0.73</td>
<td>0.83</td>
<td>0.83</td>
<td>0.74</td>
</tr>
</tbody>
</table>
Figure 17 compares the porosity predictions to the well observations. Posterior means and 0.8 prediction intervals are displayed. The prediction intervals for the mixture model are assessed by sampling from the posterior distribution. Recall that well 1 is used in global parameter inference, but wells 2–6 appear as blind tests. The predictions based on the mixture model a (red) compares well with observations (black), particularly in the reproductions of contrasts. The Gauss-linear predictions (blue) are also comparable to the observations (black), but the predictions appear smoother. Note that the predictions for both models are poorest in the nonhydrocarbon-bearing well 6. The prediction intervals have comparable width for the two models.

Table 1 summarizes Figure 17 in mean square error (MSE) and coverage for 0.8 prediction interval, the latter being the proportions of observed values that lie within the 0.8 prediction intervals. We observe that the model with lithology/fluid classes has reduced the MSE by more than 45% for wells 1–5 compared to the Gauss-linear model. For well 6, which is poorly reproduced, the Gauss-linear model is slightly better. The coverages, which should be close to 0.8, are for both models reasonably close to 0.8, which entails that the posterior prediction intervals are reliable.

From the blind tests in wells 2–6, we observe that the lithology/fluid mixture model gives predictions of elastic properties and porosity that reproduce the sharp contrasts and that have significantly smaller MSE than the linear model. We conclude that mixture models including lithology/fluid classes provide more reliable predictions than pure Gauss-linear models. Both models seem to provide reliable prediction intervals.

**CONCLUSIONS**

Seismic AVO data are inverted into lithology/fluid classes, elastic properties, and porosity. The likelihood model contains a convolutional, linearized seismic model and a depth-dependent rock-physics model. Several of the model parameters in the likelihood are considered to be stochastic. The prior model on lithology/fluid classes is a stationary Markov random field that captures horizontal continuity and vertical fluid sorting; hence, no trend is enforced through the prior model. The model appears as a hierarchical Bayesian inversion model, with the work flow displayed in Figure 18.

The model is applied in a 3D study on real seismic AVO data and well observations from the Alvheim field situated in the North Sea. In the study, seismic data and observations in well 1 are used for inference of the model parameters, whereas seismic data only are used as conditioning in the lithology/fluid elastic properties and porosity predictions. Hence, observations in five wells, wells 2–6, are used as true blind tests to evaluate the predictability of the model. The influences of depth trends in the rock-physics model and spatial lithology/fluid coupling in the prior model are also evaluated.

The following conclusions are drawn from the blind tests on wells 2–6 in the Alvheim field:

1. A formal hierarchical Bayesian inversion model, which is consistent in three dimensions with associated simulation algorithm, can provide very reliable predictions of lithology/fluid classes, elastic properties, and porosity.

2. Inclusion of global model parameters that are estimated from seismic data and observations in at least one well is important for reliable predictions. Moreover, accounting for the uncertainty in these global parameters is expected to improve uncertainty assessments.

3. Inclusion of spatial lithology/fluid coupling is crucial for identifying hydrocarbon accumulations. These accumulations are rare events, and contextual prior information is required to improve the resolution in the seismic AVO inversion.

4. Inclusion of rock-physics depth trends can significantly improve the classification between gas and oil; otherwise, most hydrocarbons are classified as gas.

5. Inclusion of lithology/fluid classes in the model for porosity prediction will significantly improve the predictions — contrasts in particular. The MSE improvement over a linear model is more than one-third.

Last, when making the final lithology/fluid and porosity predictions, both the seismic AVO data and observations in all six wells should be used in the parameter inference and as conditioning observations. It is possible to refine the prior spatial lithology/fluid model such that sand-shale heterogeneity and proportion trends can be reproduced. Moreover, permeability predictions can be made and inclusion of lithology/fluid classes makes it possible to use separate porosity/permeability relations for each lithology class. The seismic inversion presented in this study is feasible to make regularly. The computing demand for this study is about one hour on a regular workstation, the algorithm scale linear in the number of lattice cells, and can easily be implemented on a parallel processor to reduce the computation time.

**ACKNOWLEDGMENTS**

The research is a part of the Uncertainty in Reservoir Evaluation (URE) activity at the Norwegian University of Science and Technology (NTNU). We thank Hans Oddvar Augedal at Lundin Norway for valuable input on the reservoir geology of the North Sea field. Finally, we acknowledge the operator of the Alvheim licenses, Marathon Petroleum Norge, and partners ConocoPhillips Norge and Lundin Norway for permission to publish the results from this study.

**APPENDIX A**

**MODEL PARAMETER SPECIFICATION**

The rock-physics parameters are listed in Table A-1 and are in accordance with results in Holt and Fjær (2003), Mavko et al.

<table>
<thead>
<tr>
<th>Material</th>
<th>Bulk modulus $k$ (GPa)</th>
<th>Shear modulus $g$ (GPa)</th>
<th>Density $\rho$ (g/cm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quartz</td>
<td>37.0</td>
<td>44.0</td>
<td>2.65</td>
</tr>
<tr>
<td>Clay</td>
<td>25.0</td>
<td>6.0</td>
<td>2.65</td>
</tr>
<tr>
<td>Water (free)</td>
<td>2.4</td>
<td>0.0</td>
<td>1.03</td>
</tr>
<tr>
<td>Bounded water</td>
<td>4.0</td>
<td>6.0</td>
<td>1.03</td>
</tr>
<tr>
<td>Oil</td>
<td>1.0</td>
<td>0.0</td>
<td>0.71</td>
</tr>
<tr>
<td>Gas</td>
<td>0.1</td>
<td>0.0</td>
<td>0.29</td>
</tr>
</tbody>
</table>
Table A-2. Porosity/cementation depth trends prior model parameters. Expectations $\mu_{\lambda i}$ and standard deviations $\sigma_{\lambda i}$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\mu_{\lambda i}$</th>
<th>$\sigma_{\lambda i}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_{sh}^0$</td>
<td>0.4</td>
<td>0.2</td>
</tr>
<tr>
<td>$\phi_{sh}^0$</td>
<td>0.4</td>
<td>0.2</td>
</tr>
<tr>
<td>$\alpha_{sh}$</td>
<td>1.0</td>
<td>0.4 m$^{-1}$</td>
</tr>
<tr>
<td>$\alpha_{sh}$</td>
<td>0.35</td>
<td>0.15 m$^{-1}$</td>
</tr>
<tr>
<td>$k^*$</td>
<td>0.30</td>
<td>0.40 m$^{-1}$</td>
</tr>
<tr>
<td>$\varepsilon^*$</td>
<td>1950</td>
<td>50 m</td>
</tr>
</tbody>
</table>

where the ordering of lithology/fluid classes is sand-gas, sand-oil, sand-brine, and shale. The transition matrix has the stationary distribution $[0.10, 0.14, 0.28, 0.48]$, which represents the proportion of each class before adding lateral couplings in the prior model. The values used for the lateral coupling parameters are $\beta' = 0.5$, $\beta'' = 2$. If we want to estimate $P$, $\beta'$, and $\beta''$ by, for example, using a training image, we need to estimate the parameters jointly, which is a challenging problem due to an unknown normalizing constant in the discrete Markov random-field model (Besag, 1974). Thus, the parameter values above are assigned subjectively based on observations in well 1 to enforce spatial continuity of the lithology/fluid classes and to ensure gravitational ordering of the fluid classes. Note that the class proportions reflect the well proportions, which may overrepresent the hydrocarbons due to preferential location of the well.

The prior model for the covariance matrix for the seismic properties is parameterized as $\Sigma_m = \Sigma_m^0 \otimes I_{n_l}$, where $I_{n_l}$ is an $n_l \times n_l$ identity matrix and $\Sigma_m^0$ has an inverse gamma probability-density prior

$$p(\Sigma_m^0) = IG_1(\Sigma_m^0; 2, 0.1^2), \quad i \in \{1, 2, 3\},$$

which is a special case of the inverse Wishart distribution. The expected value of $\Sigma_m^0$ is $0.01^2$, and the variance is infinite and undefined.

The covariance matrix of seismic data $\Sigma_d$ is parameterized as $\Sigma_d = \Sigma_d^0 \otimes Y_d$, where $\Sigma_d^0$ is a $3 \times 3$ matrix. The correlation matrix $Y_d$ is an $n_t \times n_t$ matrix and assumed known:

$$\Sigma_d = \frac{1}{100} I_{n_t} + \frac{99}{100} WW^T,$$

where $I_{n_t}$ is an $n_t \times n_t$ identity matrix and $W$ is a normalized convolution matrix based on the wavelets. The first term in $\Sigma_d$ is assumed to be a measurement error, and the second term represents source-generated noise. Following Buland and Omre (2003a) and assuming mostly colored noise, the variance is divided between the terms so that the variance in the second term is about 100 times larger than the variance in the first term. This ensures mostly wavelet colored noise. The prior distribution of $\Sigma_d^0$ is an inverse Wishart probability density function:

$$p(\Sigma_d^0) = IW_{3n_t}(\Sigma_d^0; I_3, 5).$$

The expectation of $\Sigma_d^0$ is $I_3$ and by using five degrees of freedom, the prior will be very vague.

The wavelets are assumed known up to a multiplicative calibration parameter $c_w$, and the prior model for $c_w$ is

$$c_w \sim N_1(1, 0.5^2).$$

REFERENCES

Avseth, P., T. Mukerji, and G. Mavko, 2005, Quantitative seismic interpretation parameter values above are assigned subjectively based on observations in well 1 to enforce spatial continuity of the lithology/fluid classes and to ensure gravitational ordering of the fluid classes. Note that the class proportions reflect the well proportions, which may overrepresent the hydrocarbons due to preferential location of the well.

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