Norwegian University
of Science and Technology
Department of Mathematical Sciences

English

Contact during the exam: Professor Jarle Tufto
Phone: 99 70 55 19

Statistics with Applications, ST0103

August 10, 2013
Kl. 9–13
Grades are due September 1

Permitted aids: One handwritten A4 sheet, calculator,"Tabeller og formler i statistikk" (Tapir forlag), K. Rottmann: Matematisk formelsamling.

Give your reasoning behind all answers.

Problem 1 Let X and Y be normally distributed independent stochastic variables. Assume that X has expected value 0 and standard deviation 1, whereas Y has expected value 1 and standard deviation 2.

a) Make a sketch of the probability density functions of X and Y in a common plot.

b) Find the probabilities $P(X \leq 1.2)$, $P(Y > 2)$ and $P(X + Y \leq 2)$.

c) Find the probability density function of $Z = e^X$.

Problem 2 We position some sampling sites inside an area of a given vegetation type. This vegetation type may include the species Elm and Beech with certain probabilities. Suppose that the probability that Elm is found at a given sampling site is 0.2, that the probability that Beech is found is 0.5, and that the the total probabity that either or both species are found is 0.6. Let A denote the event that Elm is found at a given site and B the event that Beech is found.

a) What is meant by disjoint events? Are A and B disjoint events?

b) What is meant by independent events? Are A and B independent events?

Problem 3 *Ornithomyia chloropus* () is a common parasite of house sparrows. Suppose that we observe a random sample of $n = 50$ sparrows and that X out of these turns out to be parasitized. Let p be the proportion of parasitized individuals in the population and assume that the population is much larger than the sample size.

a) Which distribution would you assume that X has? What assumptions does this require?

Suppose that 45 out of the 50 sparrows in the sample are parasitized.

b) Compute a point estimate of the parameter p and an associtated approximate 95%-confidence interval.

Suppose that the number of parasites present at each individual sparrow is Poisson-distributed with parameter μ such that Y follows the distribution

$$P(Y = y) = \frac{\mu^y e^{-\mu}}{y!}. \tag{1}$$

The event that an individual is parasitized is then equivalent with the event that one or more parasites are present at a given individual, that is, the event $Y \geq 1$.

c) Find the probability of this event expressed as a function of μ.

d) Use the relationship you have found between p and μ to derive a point estimate and an approximate 95%-confidence interval for the expected number of parasites present on each sparrow. Does the answer seem reasonable?

Suppose in the following that we observe not only whether each sparrow is parasitized or not, but also the actual number of parasites $y_1, y_2, \ldots, y_{50}$ present on each bird in the sample. A total number of $\sum_{i=1}^{50} y_i = 121$ parasites are observed.

e) Use this additional information to compute a new point estimate and a 95%-confidence interval for μ based on a normal approximation. Discuss briefly whether and for what reason this new estimate or the estimate in the previous point is preferrable.