Dynamical systems in $\beta\mathbb{Z}$ and equivariant maps of $\ell^\infty(\mathbb{Z})$

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Abstract

The integers have a natural action on the Stone-Čech compactification $\beta\mathbb{Z}$ of themselves, and this leads to a dynamical system and a semigroup structure on $\beta\mathbb{Z}$. We will use this to study maps $\ell^\infty(\mathbb{Z}) \to \ell^\infty(\mathbb{Z})$ that are equivariant with respect to translation. Following work by Vern Paulsen, we will extend these to equivariant self-maps of the crossed product $\ell^\infty(\mathbb{Z}) \rtimes \mathbb{Z}$. In particular when the map corresponds to a single point ω in $\beta\mathbb{Z}$, and especially when this point is a minimal idempotent, the image of the map A_ω, which becomes a C^*-algebra, is of interest. One can show that a Laurent operator $a \in B(H)$ is pavable, i.e. satisfies the Kadison-Singer conjecture, if and only if it is contained in the injective envelope (that is a copy of it) of A_ω where ω is any minimal idempotent. Solving the KS-problem for Laurent operators is believed to be a critical step in solving the conjecture in general.