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Title

An Energy-Conserving Second Order Numerical Scheme For Nonlinear Hyperbolic Equation With An Exponential Nonlinear Term

Abstract

We present a second order accurate numerical scheme for a nonlinear hyperbolic equation with an exponential nonlinear term. The solution to such an equation is proven to have a conservative nonlinear energy. Due to the special nature of the nonlinear term, the positivity is proven to be preserved under a periodic boundary condition for the solution. For the numerical scheme, a highly nonlinear fractional term is involved, for the theoretical justification of the energy stability. We propose a linear iteration algorithm to solve this nonlinear numerical scheme. A theoretical analysis shows a contraction mapping property of such a linear iteration under a trivial constraint for the time step. We also provide a detailed convergence analysis for the second order scheme, in the $\ell^\infty(0,T;\ell^\infty)$ norm. Such an error estimate in the maximum norm can be obtained by performing a higher order consistency analysis using asymptotic expansions for the numerical solution. As a result, instead of the standard comparison between the exact and numerical solutions, an error estimate between the numerical solution and the constructed approximate solution yields an $O(\Delta t^3 + h^4)$ convergence in $\ell^\infty(0,T;\ell^2)$ norm, which leads to the necessary ℓ^∞ error estimate using the inverse inequality, under a standard constraint $\Delta t \le Ch$. A numerical accuracy check is given and some numerical simulation results are also presented.