HOCHSCHILD COHOMOLOGY OF RELATION EXTENSION ALGEBRAS

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An algebra B is said to be a split extension of an algebra C by a C-C-bimodule E if there is an exact sequence of vector spaces

$$0 \to E \xrightarrow{i} B \underset{q}{\overset{p}{\rightleftarrows}} C \to 0$$

where p is an algebra morphism which has a section q which is also an algebra morphism. Many properties of the algebras B and C have been compared in various papers.

We introduce an algebra morphism $\varphi^* : \mathrm{HH}^*(B) \to \mathrm{HH}^*(C)$ relating the Hochschild cohomologies of the two algebras. The map φ^1 was previously introduced and studied by I. Assem and M.J. Redondo. We then turn to the case when the ideal E in B satisfies $E^2 = 0$, when B is called a trivial extension of C by E. In this situation, we introduce necessary and sufficient conditions for each φ^n to be surjective and use them to prove that φ^1 is surjective when $E = \mathrm{Ext}^m_{C\text{-}C}(DC, C)$. We then apply this to relation-extensions and generalise results of I. Assem, J.-C. Bustamante, K. Igusa, M.J. Redondo and R. Schiffler when B is a cluster-tilted algebra.

This is joint work with I. Assem, M.A. Gatica and R. Schiffler.