Online workshop on

Quantum Harmonic Analysis and Applications to Operator Theory 2021

Leibniz University Hannover
and
NTNU Trondheim

August 3rd to August 5th 2021

Schedule

All times in the schedule are according to central European summer time!

Tuesday, August 3rd

15:00 - 15:15	Opening
15:15 - 16:15	Reinhard F. Werner
16:30 - 17:30	Franz Luef
17:45 - 18:45	Robert Fulsche

Wednesday, August 4th

15:00 - 15:45	Elena Cordero
16:00 - 16:45	Monika Dörfler
17:00 - 17:45	Hans Georg Feichtinger
18:00 - 18:45	Franz Luef
19:00 - 19:25	Helge Knutsen
19:30 - 19:55	Federico Bastianoni

Thursday, August 5th

15:00 - 15:45	Reinhard F. Werner
16:00 - 16:45	Jukka Kiukas
17:00 - 17:45	Raffael Hagger
18:00 - 18:45	Wolfram Bauer
19:00 - 19:25	Lauritz van Luijk

Abstracts Tuesday

Quantum Harmonic Analysis: Motivations and basic structure

Reinhard F. Werner

Leibniz University Hannover

Abstract

The original motivation for my 1984 paper came from the work of my thesis advisor Günther Ludwig on axiomatic quantum mechanics and more general statistical theories.

I will describe this context and how it led to a graded convolution algebra as the basic arena for quantum-classical connections. This is also helpful for expressing common physical intuitions about joint measurements of position and momentum, uncertainty, and counting states via phase space volume.

Localization operators

Franz Luef

NTNU Trondheim

Abstract

We describe the basics of time-frequency analysis with a focus towards localization operators and we recast this basic notion of time-frequency analysis from the perspective of quantum harmonic analysis. Furthermore, we discuss some recent developments based on this approach: mixed-state localization operators and the accumulated Cohen class of mixed-state localization operators. This is based on joint work with Eirik Skrettingland.

Correspondence Theory and Toeplitz operators on Fock spaces

Robert Fulsche

Leibniz University Hannover

Abstract

We will start by giving a short introduction to (Segal-Bargmann)-Fock spaces and their Toeplitz operators. Afterwards, we will explain how to embed these operators into the framework of Quantum Harmonic Analysis.

In particular, we will discuss the Correspondence Theorem of QHA. It will be demonstrated how this theorem can be used to obtain short proofs of some key theorems on Toeplitz operators. Finally, we will discuss how the Fock space picture can be used to obtain new structural results in the Correspondence Theory of QHA.

Abstracts Wednesday

Wigner Analysis of Operators

Elena Cordero

University of Torino

Abstract

We perform Wigner analysis of linear operators. Namely, the standard time-frequency representation *Short-time Fourier Transform* (STFT) is replaced by the $\mathcal{A}$-*Wigner distribution* defined by

$$W_{\mathcal{A}}(f) = \mu(\mathcal{A})(f \otimes \overline{f}),$$

where $\mathcal{A}$ is a $4d \times 4d$ symplectic matrix and $\mu(\mathcal{A})$ is an associate metaplectic operator. Basic examples are given by the so-called τ-Wigner distributions. Such representations provide a new characterization for modulation spaces when $\tau \in (0,1)$. Furthermore, they can be efficiently employed in the study of the off-diagonal decay for pseudodifferential operators with symbols in the Sjöstrand class (in particular, in the Hörmander class $S^0_{0,0}$).

The novelty relies on defining time-frequency representations via metaplectic operators, developing a conceptual framework and paving the way for a new understanding of quantization procedures.

Structure and effective Dimensionality of Time Series Data Sets

Monika Dörfler

University of Vienna

Abstract

For high-dimensional, complex data sets, structured dimensionality reduction methods are essential in order to enable useful further processing. While image data are directly accessible and rather easy to interpret, time series data such as audio seem to be harder to understand, cluster and classify. In our work, we apply an approach based on a generalization of time-frequency localization operators (mixed-stated localization operators), which is inspired by quantum information theory in order to get access to the intrinsic structure and effective dinemsionality of given 1D-data sets.

Why is THE Banach Gelfand Triple the natural framework for Gabor Analysis?

Hans Georg Feichtinger

University of Vienna

Abstract

In this talk the (well known) usefulness of the Banach Gelfand Triple (SO,L^2, SO*) for the discussion of questions arising naturally in Gabor Analysis (and more generally in time-frequency analysis) are summarized. Although most of the statements presented are found in a variety of different papers of even books the speakers will try to share is own experience in using this setup for the treatment of problems arising naturally in the context of Gabor Analysis.

Recall, the Segal algebra SO is the smallest (non-trivial) Banach space of continuous (and integrable) functions which are isometrically invariant

under time-frequency shifts. It is Fourier invariant and even invariant under metaplectic transformations such as the Fractional FT. The dual space, meanwhile called the space of mild distributions, can be viewed as the largest domain of “signals” which have a bounded spectrogram.

As time permits also some indication about open problems will be given. Otherwise after the conference a small collection of such questions can be circulated.

Tauberian theorems for operators

Franz Luef

NTNU Trondheim

Abstract

We present variants of Wiener’s Tauberian Theorem in the setting of quantum harmonic analysis. We also discuss Pitt’s theorem for operators and some of its applications to localization operators which are generalizations of Toeplitz operators on Bargmann-Fock spaces. Note that the analogue of Pitt’s Tauberian theorem in our setting implies compactness results for localization/Toeplitz operators in terms of the Berezin transform. This is joint work with Eirik Skrettingland.

Fractal Uncertainty Principle and Daubechies Time-Frequency Localization Operator

Helge Knutsen

NTNU Trondheim

Abstract

At the heart of time-frequency analysis lie the uncertainty principles, which all, in some form, state that a signal cannot be highly-localized in time and frequency simultaneously. Many versions of the uncertainty principles exist, and a recent version, introduced by S. Dyatlov and J. Zahl in 2016, referred to as the Fractal Uncertainty Principle (FUP), considers the concentration of a signal near fractal sets in time and frequency. I will provide a brief introduction to the FUP as stated for the separate time-frequency representation, and later present a first iteration analogue in the joint representation based on the Short-Time Fourier transform and Daubechies' classical localization operator. In particular, we shall consider operators with a Gaussian window and radially symmetric symbol localizing on a family of Cantor type sets.

Quasi-Banach Modulation Spaces and Localization Operators on LCA groups

Federico Bastianoni

Politecnico di Torino

Abstract

We define the modulation spaces $M^{p,q}_m$ on a LCA group $\mathcal{G}$ for $0 < p, q \leq \infty$. Connections and equivalences with the classical theory for $p, q \geq 1$ by Feichtinger 1983 are investigated, moreover the Euclidean, discrete and compact cases are fully recovered. We then apply this theory to the study of boundedness properties of pseudodifferential operators. Finally, decay properties for eigenfunctions of localization operators are investigated.

Abstracts Thursday

Quantum-classical hybrids and the category of quasifree operations

Reinhard F. Werner

Leibniz University Hannover

Abstract

Many fundamental processes of quantum physics involve exchange of information with classical systems. This is best expressed by describing classical and quantum parts together in one structure.

However, there is a typical mismatch in the functional analytic description of states: In the classical case one has probability measures on phase space, naturally considered as the dual of the continuous functions. In the quantum case one has density operators, naturally considered as the pre(!)dual of the bounded operators.

I will describe an approach to hybrids that resolves this tension, and allows a very smooth unified formalism. Operations naturally have both a Heisenberg picture (acting on observables) and a Schrödinger picture (acting on observables) without the need of going to unconstructive objects like the dual of the bounded operators. This is illustrated by the example of quasifree operations, widely used in quantum optics, which act by a linear transformation of (quantum or classical) phase space variables and added noise.

Joint localisation on multiple isotropic subspaces of the phase space

Jukka Kiukas

Aberystwyth University

Abstract

Convolution with a probability measure is a standard way of producing "unsharp" quantum observables from spectral measures. When applied collectively to a set of non-commuting spectral measures, convolution may render the resulting observables classical in the sense of being jointly measurable, i.e. obtainable via post-processing from a single "hidden'' observable. In the context of phase space quantum mechanics, joint measurability problems of this type are tractable even beyond the fully Gaussian setting. The starting point is the known result according to which convolutions of two non-commutative quadratures (linear combinations position and momentum) are jointly measurable only if the variances of the convolving measures satisfy the associated Heisenberg uncertainty relation. I will discuss generalisations of this result to the case of multiple quadratures and higher dimensional projective localisation observables.

QHA and the Toeplitz algebra

Raffael Hagger

University of Reading

Abstract

Let μ_α denote the Gaussian measure on $\mathbb{C}^n$ given by

$$\mathrm{d}\mu_\alpha(z) = \left(\frac{\alpha}{\pi}\right)^n \mathrm{d}z$$

and consider the corresponding L^p-space $L^p_\alpha := L^p(\mathbb{C}^n, \mu_{\alpha p/2})$ for $p \in (1, \infty)$. The closed subspace of entire functions in L^p_α is called the Segal–Bargmann

or Fock space and denoted by F^p_α. The corresponding orthogonal projection $P_\alpha : L^2_\alpha \to F^2_\alpha$ can be extended to a projection from L^p_α onto F^p_α. Of particular interest in this context are the Toeplitz operators $T_f : F^p_\alpha \to F^p_\alpha$, which are given by

$$T_f g := P_\alpha(fg)$$

for bounded symbols $f : \mathbb{C}^n \to \mathbb{C}$, and the Banach algebra generated by all Toeplitz operators with bounded symbol. For instance, a bounded linear operator on the Fock space is compact if and only if it is in the Toeplitz algebra and its so-called Berezin transform vanishes at infinity [1]. However, it is notoriously difficult to check directly whether a given operator is contained in the Toeplitz algebra. Recently, Fulsche [2] showed quite intriguingly that methods from quantum harmonic analysis can be used to study the Toeplitz algebra. In this talk I will present some direct consequences of Fulsche's work including several characterizations of the Toeplitz algebra which provide better membership criteria [3].

[1] W. Bauer, J. Isralowitz: Compactness characterization of operators in the Toeplitz algebra of the Fock space F^p_α, J. Funct. Anal. 263 (2012), 1323-1355.

[2] R. Fulsche: Correspondence theory on p-Fock spaces with applications to Toeplitz algebras, J. Funct. Anal. 279 (2020), no. 7, 108661.

[3] R. Hagger: Essential Commutants and Characterizations of the Toeplitz Algebra, J. Operat. Theor. 86 (2021), no. 1, 125-143.

Wolfram Bauer

Leibniz University Hannover

Abstract

The Classical Limit of Quantum Mechanics

Lauritz van Luijk

Leibniz University Hannover

Abstract

I will present my master thesis which is about a rigorous framework developed by Reinhard Werner for describing the Classical Limit of Quantum Mechanics.

The main ingredients are Quantum Harmonic Analysis and approximate inductive systems of C^*-algebras. I will a present a theorem on the convergence of dynamics generated by unbounded Hamiltonians.

Participants

- Federico Bastianoni, Politecnico di Torino
- Wolfram Bauer, Leibniz University Hannover
- Eirik Berge, NTNU Trondheim
- Stine Marie Berge, NTNU Trondheim
- Elena Cordero, University of Turin
- Monika Dörfler, University of Vienna
- Hans Georg Feichtinger, University of Vienna
- Robert Fulsche, Leibniz University Hannover
- Anupam Gumber, Indian Institute of Science Bangalore
- Raffael Hagger, University of Reading
- Jonathan Arturo Hernández Hermida, CINVESTAV, Mexico City
- Crispin Herrera-Yañez, ESCOM, Mexico City
- Jukka Kiukas, Aberystwyth University
- Helge Knutsen, NTNU Trondheim
- Christian Leal, National Polytechnic Institute, Mexico
- Germán Sameed López Paredes, CIMAT, Guanajuato
- Franz Luef, NTNU Trondheim
- Josué Ramírez-Ortega, Universidad Veracruzana

- Luigi Giacomo Rodino, University of Torino
- Miguel Rodriguez, Leibniz University Hannover
- Alejandro Soto González, CINVESTAV, Mexico City
- S. Ivan Trapasso, University of Genoa
- Lauritz van Luijk, Leibniz University Hannover
- Nikolai Vasilevski, CINVESTAV, Mexico City
- Reinhard F. Werner, Leibniz University Hannover