Formal Framework for Optimizing Vector Field Driven Ocean Sampling AUVs

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Motivation

Develop a Mathematical framework for the control for ocean sampling AUVs while optimizing onboard resources and using fluid flow fields to power motion.

General AUV Motion Planning and Control in a Fluid Vector Field

$$\begin{aligned} &\text{Minimize } \; g_0(x_0,x_f) + \int_{t_0}^{t_f} g(x,y,u,v)dt \\ &\text{subject to } \; \dot{x} = f(x,y,u,v), \quad (x_0,x_f) \in E \subset \mathbb{R}^{2n} \\ &\qquad \frac{\partial y}{\partial t} + \mathcal{A}(y,v)\frac{\partial y}{\partial x} = 0,\; y(t,x) = y_0(t),\; x \in \partial\mathcal{D}(t) \quad (1) \\ &\qquad x(t) \in \mathcal{D}(t),\; u(t) \in \Omega,\; v(t) \in \mathcal{V} \quad \forall t \in [t_0,t_f] \end{aligned}$$

- x and y are the state of the AUV and of the fluid vector field
- u and v are the AUV and fluid flow motion harvesting controls
- $\mathcal{D}(t)$ is some compact domain of interest

Main difficulty: Flow field dynamics - **Navier-Stokes** Eq.
Approach: either a priori known, or "easy" to compute vector fields.

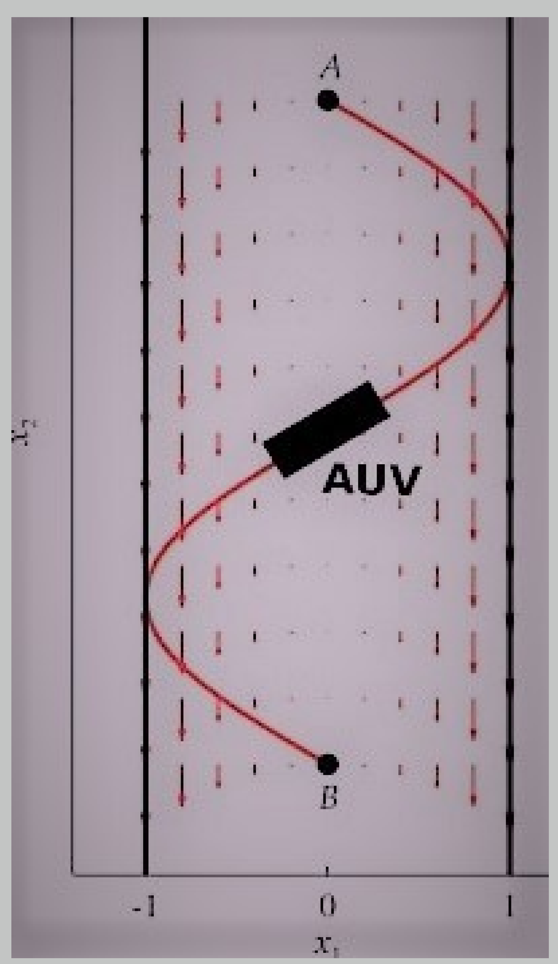
Main Results:

- Indirect method for state constrained control problems in a fluid flow, [3]
- AUV optimal control by generating point vortices (fish-like motion), [5, 4]
- Novel MPC scheme with minimal on-board motion computation burden, [2]

Indirect Method for State Constrained Control Problems

Goal: Minimum time AUV trajectory from A to B in a vector field $v(\cdot)$

$$\begin{aligned} (P)\ &\text{Minimize } T \\ &\text{subject to } \dot{x} = u + v(x) \\ &\qquad x(0) = A, \quad x(T) = B \\ &\qquad -1 \le x_1 \le 1 \\ &\qquad u = (u_1,u_2) \in U \end{aligned}$$



- **Approach**: Compute field of extremals with an indirect method using the Pontryagin Maximum Principle (PMP).
- **Challenge**: Usual PMP features merely Borel measurable measure multiplier, not amenable to numerical computations.
- **Solution**: PMP in the Gamkrelidze framework with additional **regularity assumption** implying the measure multiplier continuity, [1, 8], and define a new algorithm to solve the Boundary Value Problem (BVP).

Pontryagin Maximum Principle, [8]

Let (x^*, u^*) solve (P). Then, $\exists(\lambda, \psi, \mu)$ such that:

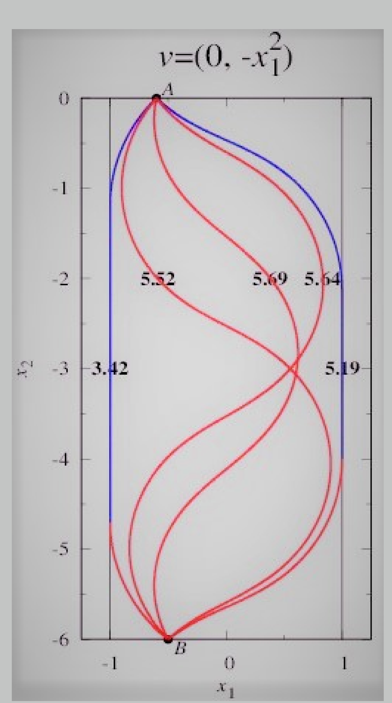
- $\dot{\psi}(t) = -\psi(t)\frac{\partial v}{\partial x} + \mu(t)\frac{\partial v_1}{\partial x}$, a.e. $t \in [0, T]$
- $u^*(t) = arg\max_{u \in U}\{(\psi_1(t) - \mu(t))u_1 + \psi_2(t)u_2\}$
- $\mu(t)$ is **continuous**, increasing if $x_1^*(t) = -1$, decreasing if $x_1^*(t) = 1$, and constant otherwise
- $\lambda + |\psi_1(t) - \mu(t)| + |\psi_2(t)| > 0 \;\; \forall t \in [0, T]$.

Algorithm and Simulation results

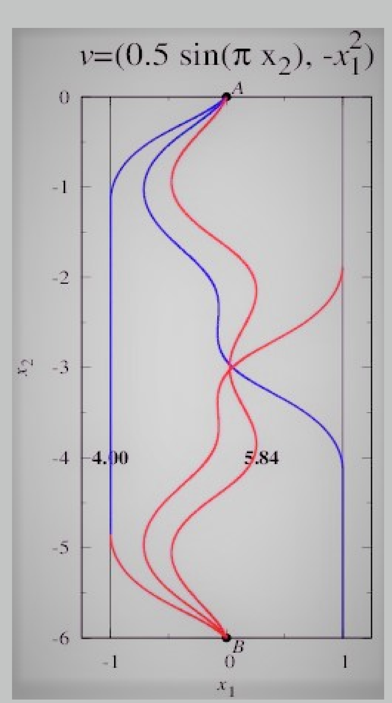
The Algorihm Solve the BVP - primal and dual DE - with multiple initial conditions for ψ:

- **backward from B** to get the "red" trajectories
- **forward from A** to get "blue" trajectories

$\mu(t)$ is continuous at the boundary, and if forward and backward trajectories meet there, the continuity of $\mu(t)$ is checked at the meeting point.



A and B are closer to the left boundary. Trajectory in the left is optimal.



Transversal component $v_1 \neq 0$. Trajectories not meeting are not extremals.

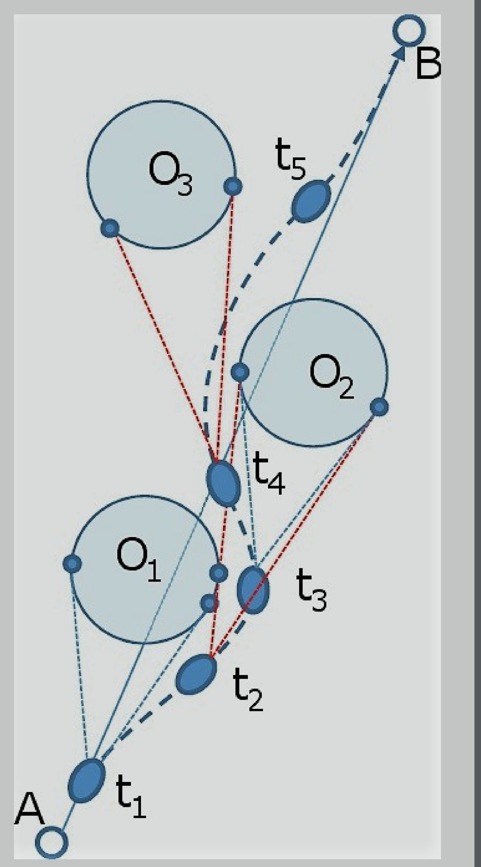
Low On-line Computational Cost MPC[2]

Key Idea of Attainable Set-MPC: Use time invariance of data to maximize off-line computation.

- [1.] Initialization: $t = t^0$, $\bar{x} = x(t_0)$
- [2.] Compute x^* and $u^*_{|[t_0,t_0+\Delta]}$ by solving

$$\begin{aligned} &\text{Minimize } \; \tilde{V}(t_0+\Delta, z) \\ &\text{subject to } \; z \in \tilde{\mathcal{A}}_f(t_0+\Delta; t_0, \bar{x})) \end{aligned}$$

- [3.] Apply u^* during $[t_0, t_0+\Delta]$
- [4.] Obtain $\bar{x}$ by sampling x at $t_0+\Delta$
- [5.] Update parameters with on-line identification methods
- [6.] Slide time by Δ, i.e., $t_0 = t_0 + \Delta$, and goto **2.**



$\tilde{V}(t_0+\Delta, z)$ and $\tilde{\mathcal{A}}_f(t_0+\Delta; t_0, \bar{x}))$ are **Value Function** and **Attainable Set** discrete approximations stored in suitable on-board look-up tables[**?**]

Future Goal: MPC scheme to track fronts

Motivation

- Applications: Ocean phenomena, foreign invasive species, forest fires, pollution spills
- Computational perspective: Modelling fronts separating fluids with different properties and dynamics is usually easier than their underlying complex dynamics

Approach Outline

- **Phase 1: Off-line Front Identification**
  Abstract Inverse Problem: Find $\mathcal{D}(t)$ such that $\mathcal{A}_t(u) = g(t)$ where $u(x) = a$ if $x \in \mathcal{D}(t)$ and $u(x) = b$ if $x \notin \mathcal{D}(t)$.
  Approach: Find $\Phi : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}$ s.t. $\partial\mathcal{D}(t) = \{x : \Phi(x,t) = 0$.
  Variational methods applied to solving $\min_{\mathcal{D}(t)} \|\mathcal{A}_t(u) - g(t)\|^2$ lead to the Hamilton-Jacobi Eq. (HJE)
  $$\frac{\partial\Phi}{\partial t} = \left(J(u)^T(\mathcal{A}_t(u) - g(t))\right)|\nabla\Phi|, \quad \Phi(x,t_0) = \Phi_0(x)$$
  Level-Set methods developed by Osher and Sethian for problems involving motion surfaces or lines satisfying HJE have the ability to track the motion through topological changes[7, 6].
- **Phase 2: On-Line Identification**
  - Combine off-line identification algorithms with an MPC scheme.
  - Multiple AUVs might be control coordinated to implement a SLAM-like scheme enabling the simultaneous front mapping and tracking.

References

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Acknowledgments

Partial supports from: FCT R&D Unit SYSTEC - POCI-01-0145-FEDER-006933/SYSTEC funded by ERDF — COMPETE2020 — FCT/MEC — PT2020 - extension to 2018, Project STRIDE - NORTE-01-0145-FEDER-000033, funded by ERDF — NORTE 2020, and project MAGIC - POCI-01-0145- FEDER-032485 - funded by FEDER via COMPETE 2020 - POCI, and by FCT/MCTES via PIDDAC.