Formal Framework for Optimizing Vector Field Driven Ocean Sampling AUVs

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Motivation

Develop a mathematical framework for the control of ocean sampling AUVs while optimizing onboard resources and using fluid flow fields to power motion.

General AUV Motion Planning and Control in a Fluid Vector Field

Minimize \( g(x_0, x_f) + \int_0^T g(x, y, u, v) \, dt \)
subject to \( \dot{x} = f(x, y, u, v), \quad (x_0, x_f) \in E \subset \mathbb{R}^n \)
\[ \frac{\partial y}{\partial t} + A(y, v) \frac{\partial y}{\partial x} = 0, \quad y(t, x) = y_0(x), \quad x \in D(t) \] (1)
\[ x(t_1) = x_0, \quad u(t) \in \Omega, \quad v(t) \in V \quad \forall t \in [t_0, t_f] \]

\( t \) and \( x \) are the state of the AUV and of the fluid vector field
\( u \) and \( v \) are the AUV and fluid flow motion harvesting controls
\( D(t) \) is some compact domain of interest

Main difficulty: Flow field dynamics - Navier-Stokes Eq.
Approach: either a priori known, or "easy" to compute vector fields.

Main Results:

- Indirect method for state constrained control problems in a fluid flow, [3]
- AUV optimal control by generating point vortices (fish-like motion), [5, 4]
- Novel MPC scheme with minimal on-board motion computation burden, [2]

Indirect Method for State Constrained Control Problems

Goal: Minimum time AUV trajectory from \( A \) to \( B \) in a vector field \( v(\cdot) \)

\((P)\) Minimize \( T \)
subject to \( \dot{x} = u + v(x) \)
\[ x(0) = A, \quad x(T) = B \]
\[-1 \leq x_1 \leq 1 \]
u \( (u_1, u_2) \in U \)
Approach: Compute field of extremals with an indirect method using the Pontryagin Maximum Principle (PMP)

Challenge: Usual PMP features merely Borel measurable measure multiplier, not amenable to numerical computations.

Solution: PMP in the Gamkrelidze framework with additional regularity assumption implying the measure multiplier continuity, [1, 8], and define a new algorithm to solve the Boundary Value Problem (BVP).

Pontryagin Maximum Principle, [8]

Let \((x^*, u^*)\) solve \((P)_n\). Then, \( \psi(\lambda, \psi, \mu) \) such that:
\[ \psi(t) = -\psi(t) \mu(t) + \mu(t) v(t, \mu(t)) \quad \text{a.e.} \quad t \in [0, T] \]
\[ u^*(t) = \text{arg max} \{ \psi(t) - \mu(t) u_1 + \psi(t) u_2 \} \]
\( \mu(t) \) is continuous, increasing if \( x_1(t) = -1 \), decreasing if \( x_1(t) = 1 \), and constant otherwise
\[ \lambda + |\psi_1(t)| - \mu(t) + |\psi_2(t)| > 0 \quad \forall t \in [0, T] \]

The Algorithm Solve the BVP - primal and dual DE - with multiple initial conditions for \( \psi^0 \):
- backward from \( B \) to get the "red" trajectories
- forward from \( A \) to get "blue" trajectories
\( \mu(t) \) is continuous at the boundary, and if forward and backward trajectories meet there, the continuity of \( \mu(t) \) is checked at the meeting point.

A and B are closer to the left boundary. Trajectory in the left is optimal.
Transversal component \( v \neq 0 \). Trajectories not meeting are not extremals.

Low On-line Computational Cost MPC [2]

Key Idea of Attainable Set-MPC: Use time invariance of data to maximize off-line computation.

- [1.] Initialization: \( t = t_0, x = x(t_0) \)
- [2.] Compute \( u^* \) and \( \mu^{(t_0+\Delta t)} \) by solving
Minimize \( \tilde{V}(t_0 + \Delta, z) \)
subject to \( z \in \tilde{X}(t_0 + \Delta; t_0, x(\cdot)) \)
- [3.] Apply \( u^* \) during \([t_0, t_0 + \Delta]\)
- [4.] Obtain \( x \) by sampling \( x \) at \( t = t_0 + \Delta \)
- [5.] Update parameters with on-line identification methods
- [6.] Slide time by \( \Delta \), i.e., \( t_0 = t_0 + \Delta \), and goto 2.

\( \tilde{V}(t_0 + \Delta, z) \) and \( \tilde{X}(t_0 + \Delta; t_0, x(\cdot)) \) are Value Function and Attainable Set discrete approximations stored in suitable on-board look-up tables?

Future Goal: MPC scheme to track fronts

Motivation:
- Applications: Ocean phenomena, foreign invasive species, forest fires, pollution spills
- Computational perspective: Modelling fronts separating fluids with different properties and dynamics is usually easier than their underlying complex dynamics

Approach Outline

Phase 1: Off-line Front Identification
- Abstract Inverse Problem: Find \( D(t) \) such that \( \tilde{X}(t, u) = g(t) \) where \( u(x) = a \) if \( x \in D(t) \) and \( u(x) = b \) if \( x \notin D(t) \).
- Approach: Find \( \phi : \mathbb{R}^2 \rightarrow \mathbb{R} \) s.t. \( D(t) = \{ x : \phi(x, t) = 0 \} \).
- Variational methods applied to solving \( \min \tilde{X}(t, u) \), leading to the Hamilton-Jacobi Eq.

Level Set methods developed by Osher and Sethian for problems involving motion surfaces or lines satisfying HJE have the ability to track the motion through topological changes [7, 6].

Phase 2: On-line Identification
- Combine off-line identification algorithms with an MPC scheme.
- Multiple AUVs might be control coordinated to implement a SLAM-like scheme enabling the simultaneous front mapping and tracking.

References


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