Exercises on C^*-simplicity

1) A ~~repr~~ unitary representation $\pi : \Gamma \to \mathcal{B}(\mathcal{H})$ has almost invariant vectors if there is a net of unit vectors $(\xi_i)_{i \in I}$ in $\mathcal{H}$ st
$\forall g \in \Gamma : \quad \| \pi(g)\xi_i - \xi_i \|_2 \to 0$

Assume that λ_Γ has almost inv. vectors. Prove that there is a $*$-homomorphism
$\varepsilon : C^*_{red}\Gamma \to \mathbb{C}$ st $\varepsilon(u_g) = 1$ f.a. $g \in \Gamma$.

2) Show that for a discrete group Γ the following statements are equivalent:
   a) Γ is C^*-simple
   b) If $\pi \prec \lambda$, then $\lambda \prec \pi$.

3) A discrete group Γ is called Powers group if the following statement holds:
$\forall F \subseteq \Gamma \setminus \{e\}$ finite $\exists C, D \subseteq \Gamma$ $\exists g_1, \dots, g_n \in \Gamma$:
   a) $\Gamma = C \sqcup D$ (disjoint union)
   b) $\forall g \in F : \; gC \cap C = \emptyset$
   c) $\forall i \in \{1, \dots, n\}$: $g_1 D, \dots, g_n D$ are pw. disjoint.

Show that a Powers group admits a top. free boundary.