Titles and Abstracts - Nordfjordeid Summer School 2019

July 01-05, 2019

Below there are the titles and abstracts for all the talks taking place in the summer school "Analysis, Geometry and PDE" in Nordfjordeid July 2019. The main lectures are listed first, and then the short talks are listed in a separate section. Both lists are alphabetically ordered.

Main Lectures

Franz Luef (Norwegian University of Science and Technology, Norway)

"Quantum Harmonic Analysis and its Applications"

Quantum harmonic analysis on phase space was developed by the physicist R. F. Werner in his work on the mathematical structures underlying quantum mechanics. We report on joint work with Eirik Skrettingland where we revisited Werner's work and were able to link it with concepts from mathematics and signal analysis. The mini course covers some aspects of this research.

Here is a more detailed description of the lectures.

Lecture 1: We set the stage by reviewing the Schröedinger representation of the Heisenberg group and basics of the theory of Schatten classes and why one might view Schatten classes as noncommutative ℓ^p-spaces. We continue with the definition of the convolution between a function and a trace class operator and the convolution of two trace class operators. Computation of these convolutions for rank-one operators shows that these convolutions give well-known objects in time-frequency analysis: (i) localization operators and (ii) Berezin transform. There are Fourier transforms that turn these convolutions into the products of the respective Fourier transforms.

Lecture 2: In this lecture we continue our discussion of the relation between quantum harmonic analysis and time-frequency analysis by the introduction of mixed-state localization operators and the study of some of their properties and the Hausdorff-Young-Werner inequality and its relation to Lieb's inequality for short-time Fourier transforms. We treat the pillar of quantum harmonic analysis: Tauberian theorems for the convolution between a function and a Schatten class operator. Consequences to mixed state localization operators are discussed and we remark on applications to time-frequency analysis.

Lecture 3: We continue with a treatment of Cohen class distributions from the perspective of quantum harmonic analysis and of the generalized phase space representations introduced by Klauder-Skagerstam. In the discussion we are going to define modulation spaces, a class of function spaces in terms of the matrix coefficients of the Schroedinger representation, and mention some of their properties.

Lecture 4: This lecture will focus on the accumulated Cohen class distribution which is a generalization of the accumulated spectrogram.

Didier Pilod (University of Bergen, Norway)

"On the Stability of Solitary Waves for the Generalized KdV equation"

In this mini course, we will give a detailed proof of the orbital stability of solitary waves for nonlinear dispersive equations. We will focus on the simpler case of the generalized Korteweg-de Vries equations, covering the results of Benjamin [1], Bona [2], Bona, Souganidis and Strauss [3]. The proof will be based on the modulational stability theory introduced by Weinstein [5] and will follow the nice survey by Muñoz [4]. If time allows, we will also sketch the proof of the orbital stability for the nonlinear Schr̈dinger equation. The course aims to be self-contained (outside of basic spectral theory tools).

[1] T. B. Benjamin, *The stability of solitary waves,* Proc. Roy. Soc. (London) Ser. A, **328** (1972), 153-183.

[2] J. L. Bona, *On the stability theory of solitary waves*, Proc. Roy. Soc. London Ser. A, **344** (1975), 363-374.

[3] J. L. Bona, P.E. Souganidis and W. A. Strauss, *Stability and instability of solitary waves of Korteweg-de Vries type,* Proc. Roy. Soc. London Ser. A, **411** (1987), 395-412.

[4] C. Muñoz, *Stability of integrable and nonintegrable structures,* Adv. Differential Equations **19** (2014), no. 9-10, 947-996.

[5] M. I. Weinstein, *Modulational stability of ground states of nonlinear Schrödinger equations,* SIAM J. Math. Anal., **16** (1985), no. 3, 472-491.

Sven Raum (Stockholm University, Sweden)

"Group C-algebras: Classical and Modern"*

This series of lectures provides an introduction to modern questions in group C*-algebras, notably C*-simplicity and C*-superrigidity. We start with a lecture introducing the necessary functional analytic background, before we introduce C*-algebras. We discuss fundamental results such as the Gelfand-Neimark theorem, providing an equivalence between locally compact Hausdorff spaces and abelian C*-algebras, or the all important GNS-construction. In a third lecture we discuss a sample of classical results on group operator algebras relating to type I groups and analytic group theory. The last lecture leads us to one contemporary topics in the field such as C*-simplicity or C*-superrigidity. The lectures are accompanied by two sessions of practical work.

Dennis The (The Arctic University of Norway, Norway)

"Symmetry Gaps for Geometric Structures"

For a given type of differential geometric structure, there is often a gap between the maximal and "submaximal" infinitesimal symmetry dimensions. This was first observed in the 19th century for Riemannian metrics and such symmetry gaps were subsequently classified for various other geometric structures on a case-by-case basis. The aim of this mini-course is to discuss techniques that led to a uniform approach to the symmetry gap problem for the class of parabolic geometries. This is a diverse class of geometric structures that include conformal, projective, CR, 2nd order ODE systems, and large classes of generic distributions. A priori, submaximally symmetric structures need not even be homogeneous, but remarkably, in many cases this geometric problem reduces ultimately to Dynkin diagram combinatorics, and some submaximally symmetric models can be "immediately" found (in a sense that I will make precise). (Joint work with Boris Kruglikov.)

Short Talks

Alexander Bufetov (The French National Centre for Scientific Research, France, Steklov Mathematical Institute of Russian Academy of Sciences and Institute for Information Transmission Problems, Russia)

"Extrapolation for point processes"

Consider a Gaussian Analytic Function on the disk, that is, a random series whose coefficients are independent complex Gaussians. In joint work with Yanqi Qiu and Alexander Shamov, we show that the zero set of a Gaussian Analytic Function is a uniqueness set for the Bergman space on the disk: in other words, almost surely, there does not exist a nonzero square-integrable holomorphic function having these zeros. The key role in our argument is played by the determinantal structure of the zeros given by the Peres-Virág Theorem, and we prove, in general, that the family of reproducing kernels along a realization of a determinantal point process generates the whole ambient Hilbert space, thus settling a conjecture of Lyons and Peres. In a sequel paper, joint with Yanqi Qiu, we study how to recover a holomorphic function from its values on our random set. The talk is based on the preprints arXiv:1806.02306, arXiv:1612.06751, arXiv:1605.01400

Ioannis Chrysikos (University of Hradec Králové, Faculty of Science, Szech Republic)

"Homogeneous 8-manifolds admitting invariant Spin(7)*-structures"*

In this talk we discuss the classification of canonical presentations of compact simply connected homogeneous 8-manifolds $M = G/H$ of a compact simply connected Lie group G, which admit invariant Spin(7)-structures. This is a joint work with D. Alekseevsky, A. Fino and A. Raffero.

Ulrik Bo Rufus Enstad (University of Oslo, Norway)

"Hilbert C^*-module frames in harmonic analysis"

A Hilbert C^*-module over a C^*-algebra A can be thought of as a Hilbert space where the inner product takes values in A rather than the complex numbers. These modules have found applications in harmonic analysis, in particular in wavelet theory and Gabor analysis. The notion of frames for

Hilbert spaces can be readily generalized to the Hilbert C^*-module setting. In this talk, we will see how module frames in the appropriate modules give wavelet frames and Gabor frames. In certain cases, when the C^*-algebra A is commutative, we will see that the module frames appear as continuous sections of vector bundles over the n-torus.

Nikita Evseev (Novosibirsk State University, Russia)

"Sobolev classes of functions valued in a monotone family of Banach spaces"

We study the class of Sobolev functions from a one-dimensional interval into a measurable family of Banach spaces. Namely, the class is closely related to the L^p-direct integral. In order to define weak derivatives, we introduce a notion of local Bochner integral. The basic properties of Sobolev functions are revised. In particular, we provide the characterization of Sobolev functions in terms of an upper gradient.

Gabriel Favre (Stockholm University, Sweden)

"On locally compact groups of type I"

I will recall the construction of the (maximal) group C^*-algebra and use it to give examples of group properties one can study through its corresponding group algebra. We are particularly interested in type I groups, which roughly speaking have a nice representation theory, corresponding to GCR algebras.

Valentin Lychagin (Moscow Institute of Control Systems, Russia)

"On geometrical structures, associated with differential operators"

In this talk we'll discuss various geometrical structures, associated with linear differential operators. To make this talk more concrete, we'll discuss, in detail, the case of operators acting in line bundles over 2-dimensional manifolds and having the second or the third order. We'll show how these structures, especially various connections and associated with them quantizations, allow us to get possible classifications of operators.

Petter Nyland (Norwegian University of Science and Technology, Norway)

"Matuis Conjectures for Étale Groupoids"

Around 2015, Hiroki Matui introduced two conjectures concerning minimal étale groupoids over Cantor spaces. Such groupoids serve as models for a great many unital simple C^*-algebras. The HK-conjecture relates the homology groups of the groupoid with the K-groups of its reduced groupoid C^*-algebra. Whereas the AH-conjecture relates the homology groups of the groupoid with (the abelianization of) its topological full group. I will give an introduction to these two conjectures, report on their current status, and, if time permits, say a few words of my own research on the AH-conjecture.

Sanaz Pooya (Stockholm University, Sweden)

"Simplicity and the unique trace property of certain L^p-operator algebras"

It is known that a C^*-algebra can be viewed as a closed self adjoint subalgebra of bounded operators on some Hilbert space (i.e. an L^2-space). Allowing other L^p-spaces, where $p \in [1, \infty]$ Phillips introduced the notion of L^p-operator algebras generalising C^*-algebras. Such an algebra is a closed sub-algebra of bounded operators on some L^p-space. These operator algebras which look similar to C^*-algebras, however not self adjoint, can behave similar or very different in comparison to C^*-algebras. In this talk, I will present one aspect of similarity: for $p \in (1, \infty)$, the L^p-operator algebras constructed from free groups are simple and have a unique trace. The C^*-algebraic version of this result was obtained in the mid 1970's by Powers. This is a joint work with S. Hejazian.

Eivind Schneider (University of Hradec Králové, Czech Republic)

"Differential invariants of Kundt waves"

Kundt waves are special Lorentzian spacetimes with vanishing polynomial scalar curvature invariants, meaning that they can not be distinguished by the normal methods. We start by writing the four-dimensional Kundt waves in special coordinates, so that the metric takes a particularly simple form, depending on one function of three variables. Then we solve the equivalence problem for metrics of this form by computing generators for the algebra of rational differential invariants under the Lie pseudogroup preserving the form of the metric. Joint work with Boris Kruglikov and David McNutt.

Boris Shapiro (University of Stockholm, Sweden)

"Around Bochner-Krall problem"

In 1929 S. Bochner posed the following problem. Given a linear ordinary differential operator

$$T = Q_k(x)d^k/dx^k + Q_{k-1}(x)d^{k-1}/dx^{k-1} + \ldots + Q_0(x)$$

with polynomial coefficients $Q_j(x)$ satisfying the degree condition $\deg Q_j(x) \leq j$, does the sequence of eigenpolynomials of T (which always exists due to the latter assumption on the degrees of coefficients) consists of orthogonal polynomials with respect to some real measure on $\mathbb{R}$. This problem created a new research area in mathematics with several hundreds publications. I will summarise the present situation and explain some new developments.

Francesca Tripaldi (University of Jyväskylä, Finland)

"$\ell^{q,1}$ forms on Heisenberg groups"

By definition, the $\ell^{q,p}$ cohomology of a bounded geometry Riemannian manifold is the $\ell^{q,p}$ cohomology of every bounded geometry simplicial complex quasiisometric to it. Following a result by Pansu and Rumin, we know that every $\ell^{q,p}$ cohomology class of a contractible Lie group can be represented by a form ω which belongs to L^p as well as an arbitrary finite number of its derivatives. If the class vanishes, then there exists a primitive φ of ω which belongs to L^q as well as an arbitrary finite number of its derivatives. This holds for all $1 \leq p \leq q \leq \infty$. Recent results in collaboration with Pansu show the sharp range of values of q integer for which the $\ell^{q,1}$ cohomology of the Heisenberg groups vanishes.