Key Exchange Over the Wiretap Channel

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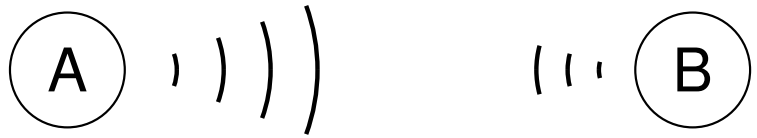
Plan

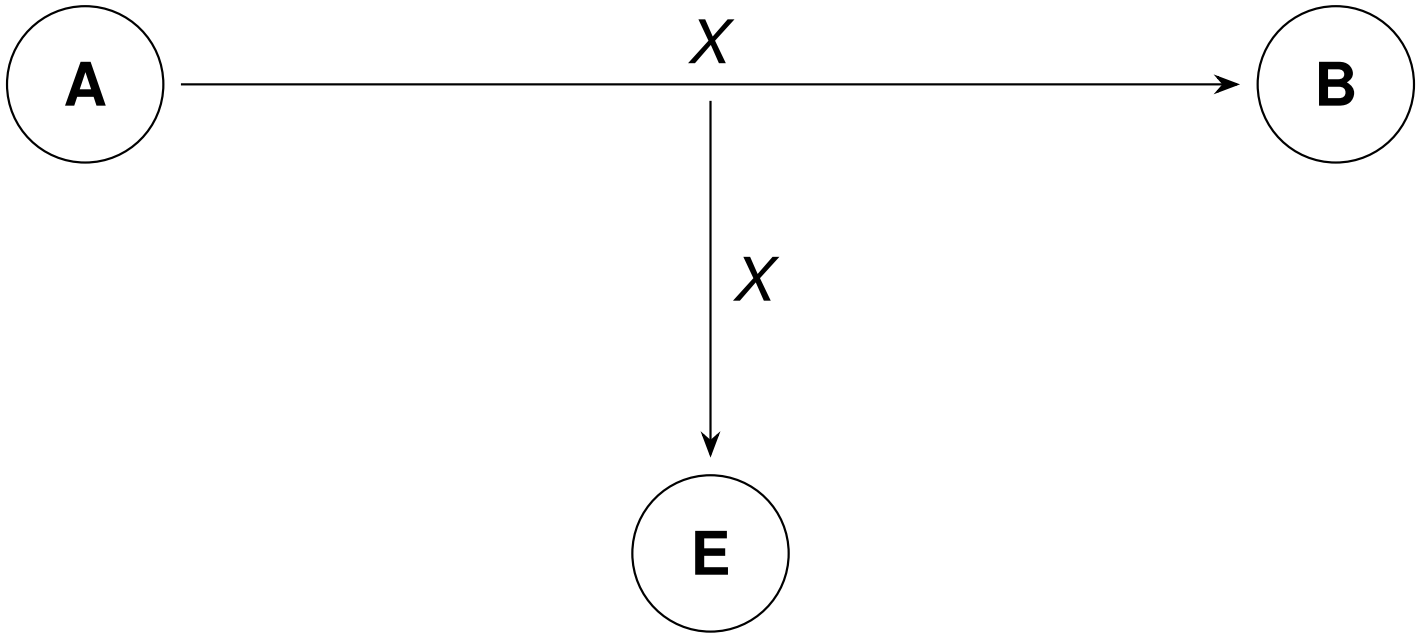
1. Motivation
2. The wiretap channel
3. Quantifying Secrecy
4. Key exchange model

Expensive Asymmetric Crypto

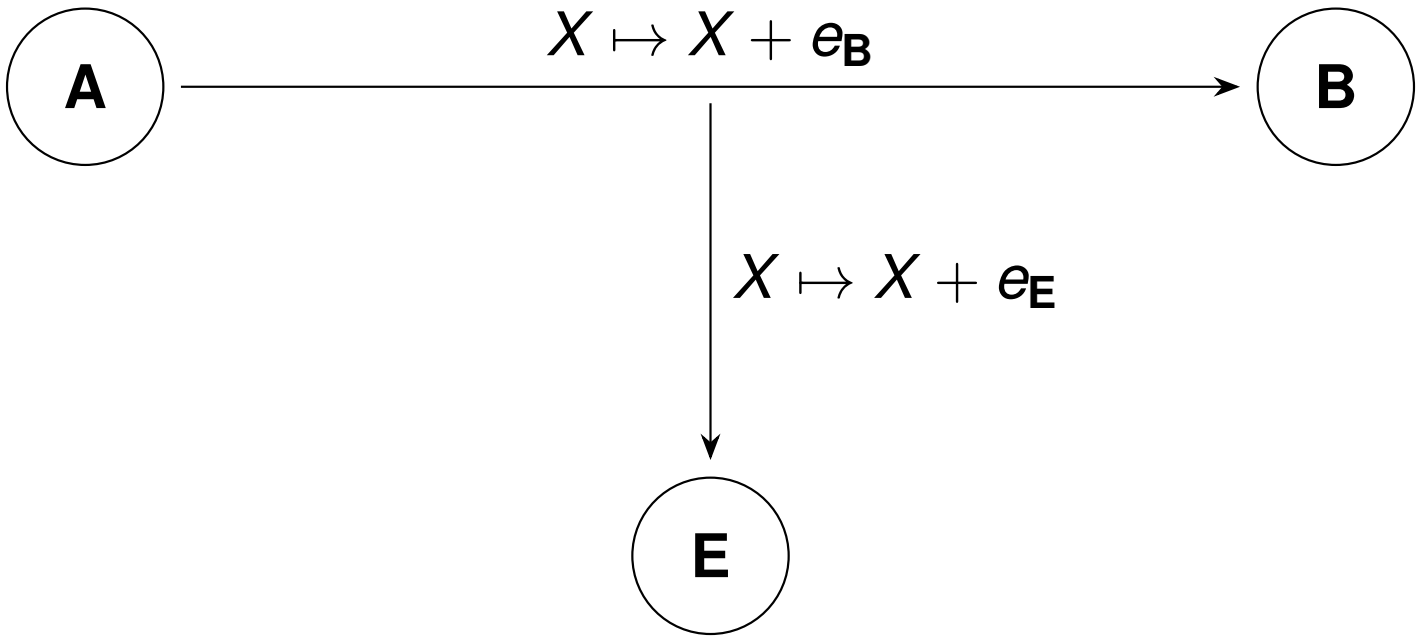
- DH is expensive for very small devices
- All asymmetric crypto is expensive
- Without asymmetric information, how can we get keys?



Channel in Standard Crypto



The Wiretap Channel



— $e_{\mathbf{E}} > e_{\mathbf{B}}$ (hopefully).

— Errors from bit-flips with probability $p_{\mathbf{B}}, p_{\mathbf{E}}$ (independently).

Paradox

— Keys *must* be equal.
— Both channels are noisy.
— Error correcting codes ruin the model.

Toy example

$p_{\mathbf{B}} = 0$ and $p_{\mathbf{E}} = 0.5$

1. Alice: $X \leftarrow \{0,1\}^{100}$
2. Alice: Sends X to Bob.
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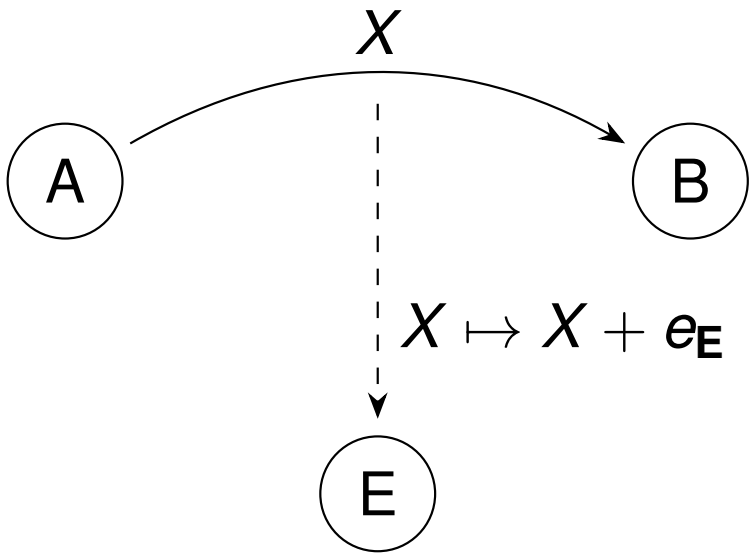

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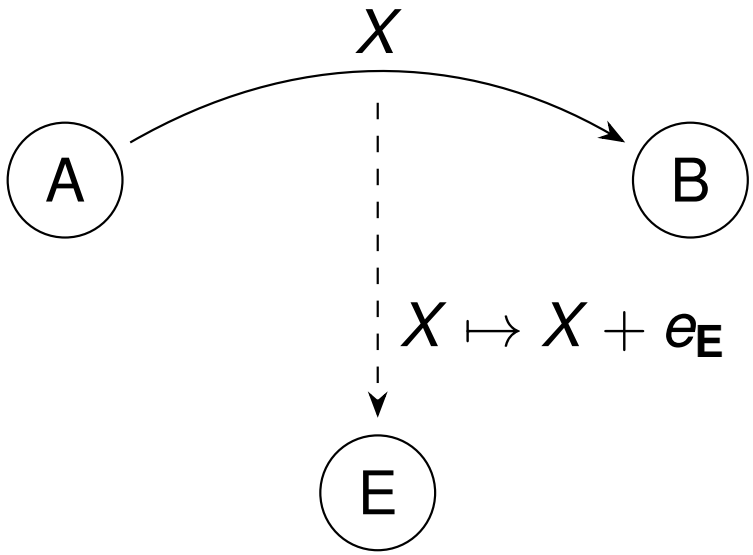

— Eve observes $X + e_{\mathbf{E}}$ independent of X.

— Eve cannot say anything about X given $X + e_{\mathbf{E}}$.

— Send 1 message $\Rightarrow$ 100 secret bits.

— Only 50% bits are different, but *all* are secret.

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— $H(X|Z) = \sum_z H(X|Z = z)\Pr[Z = z]$

Mutual information

$$I(X;Z) = H(X) - H(X|Z).$$

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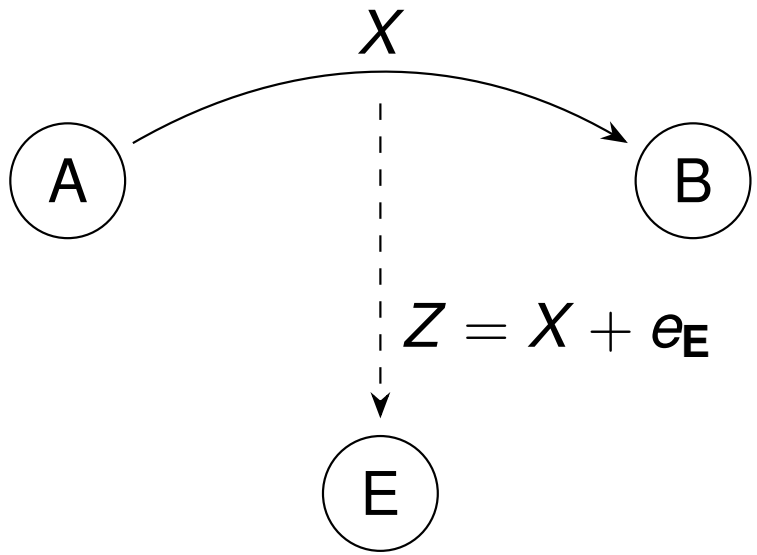

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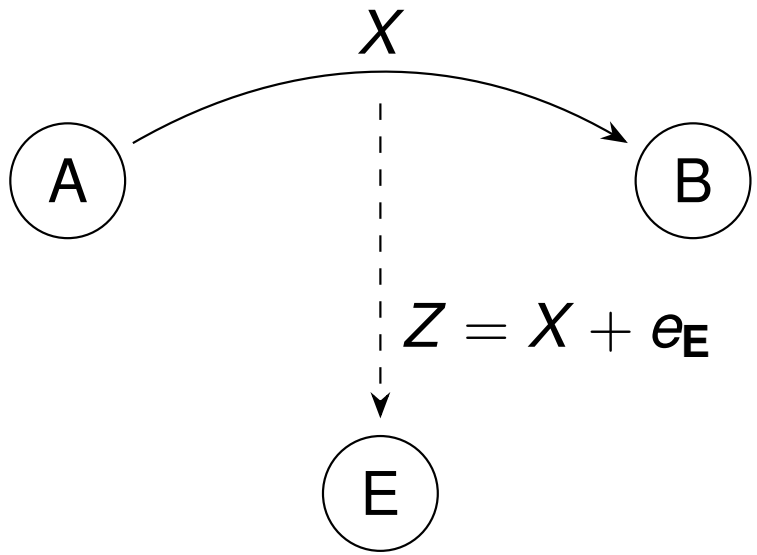

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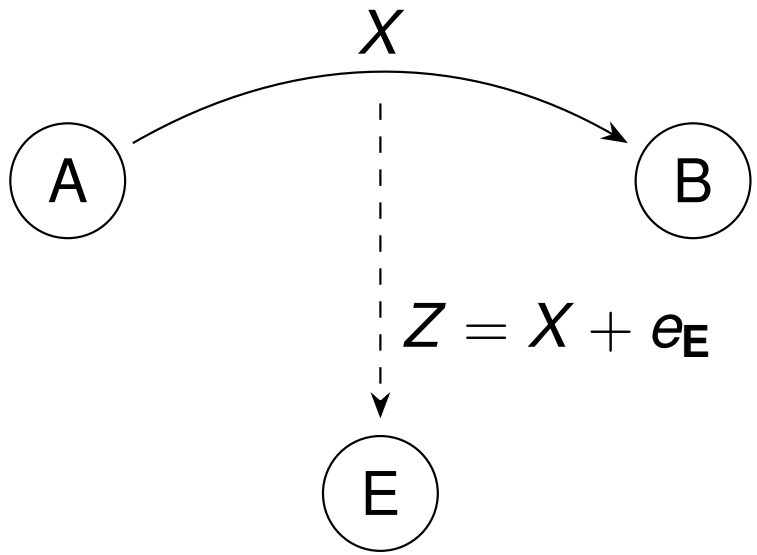

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— Eve knows nothing $\Rightarrow$ information theoretic security.

If Alice uses these bits, the problem is computational.

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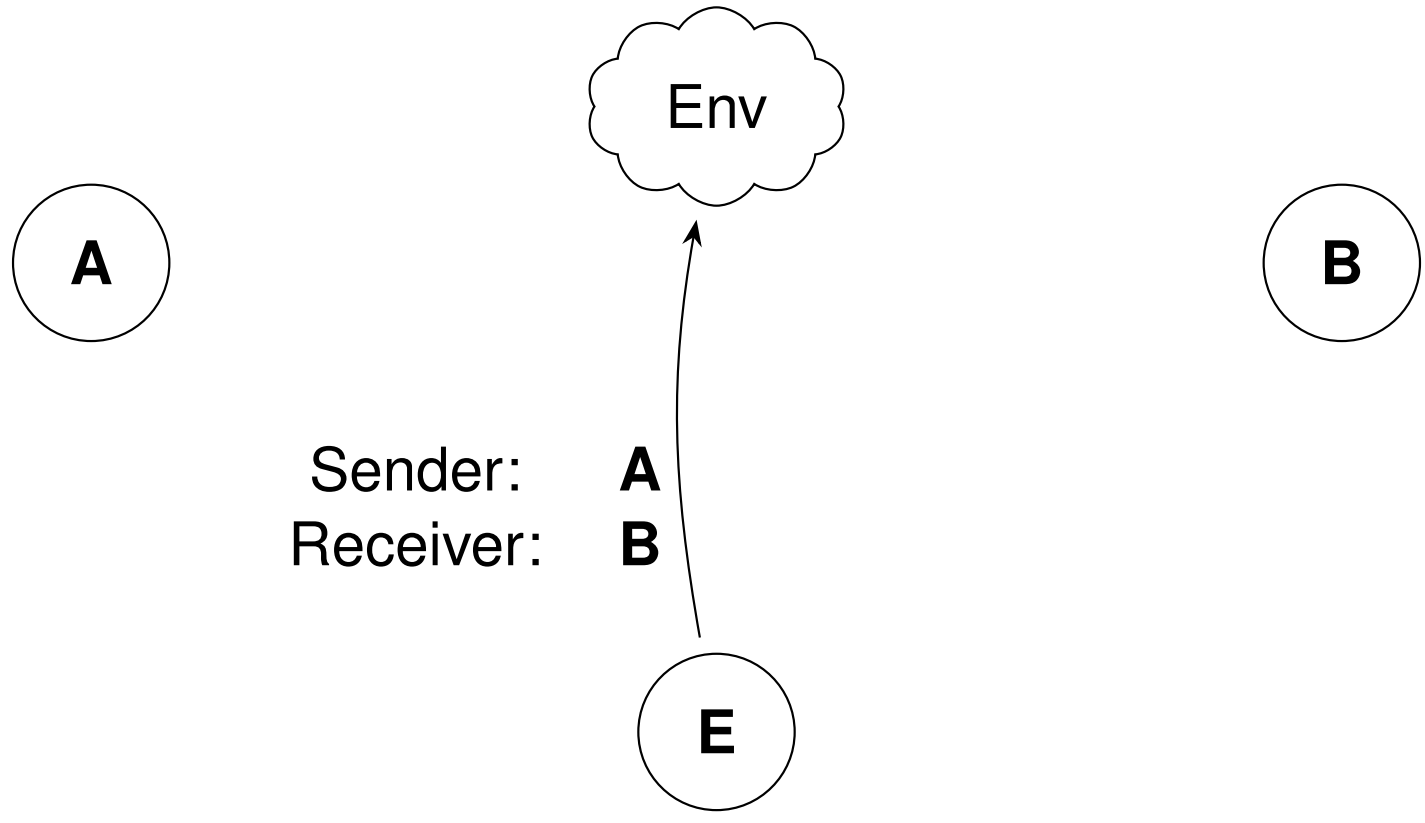
2. Use *error reconciliation*
   - Alice and Bob exchange messages over a clear channel.
   - They find where their string differ and correct errors.
   - Leaks a portion of the shared message.
   - Requires $p_{\mathbf{E}} - p_{\mathbf{B}}$ above some threshold.

Security model



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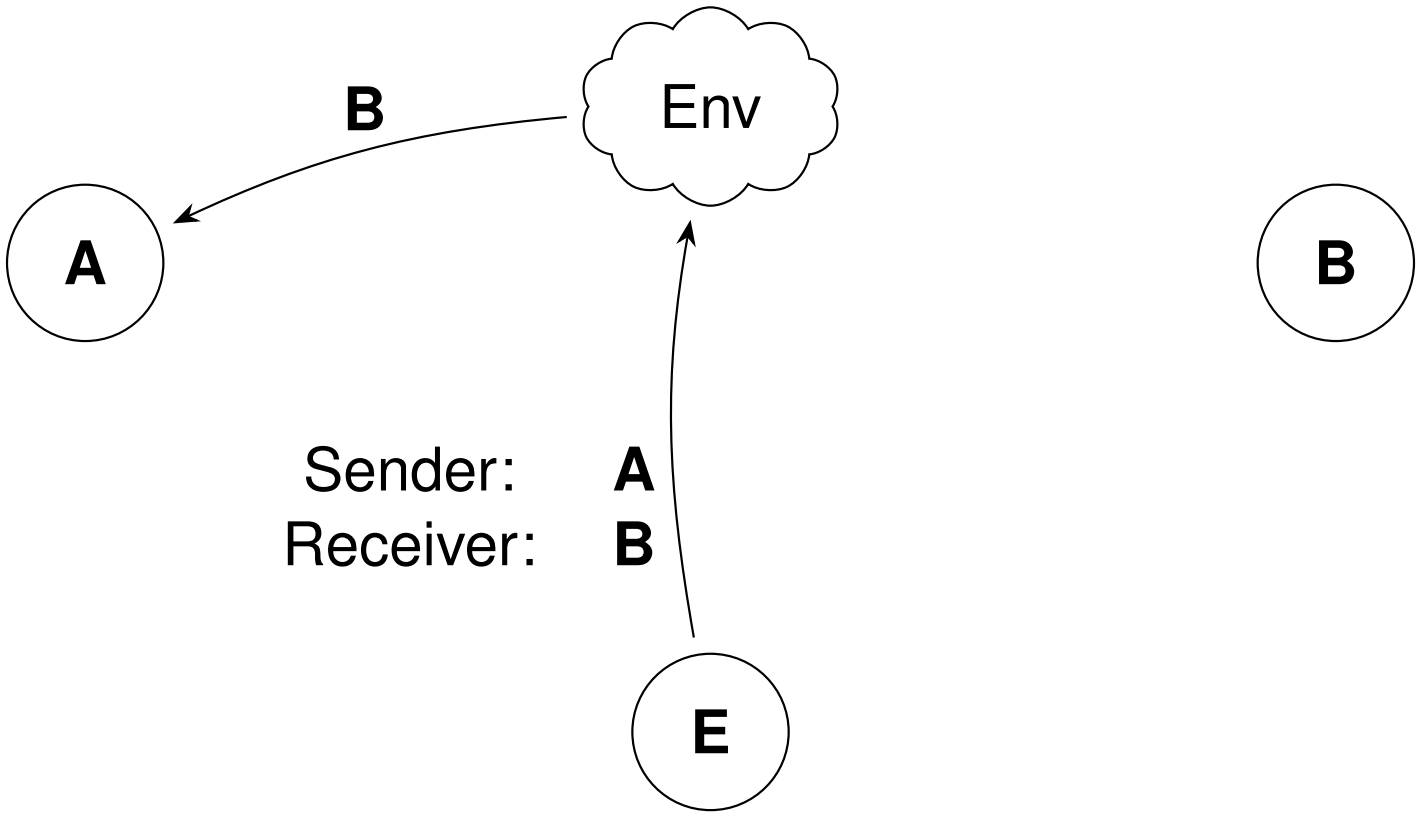

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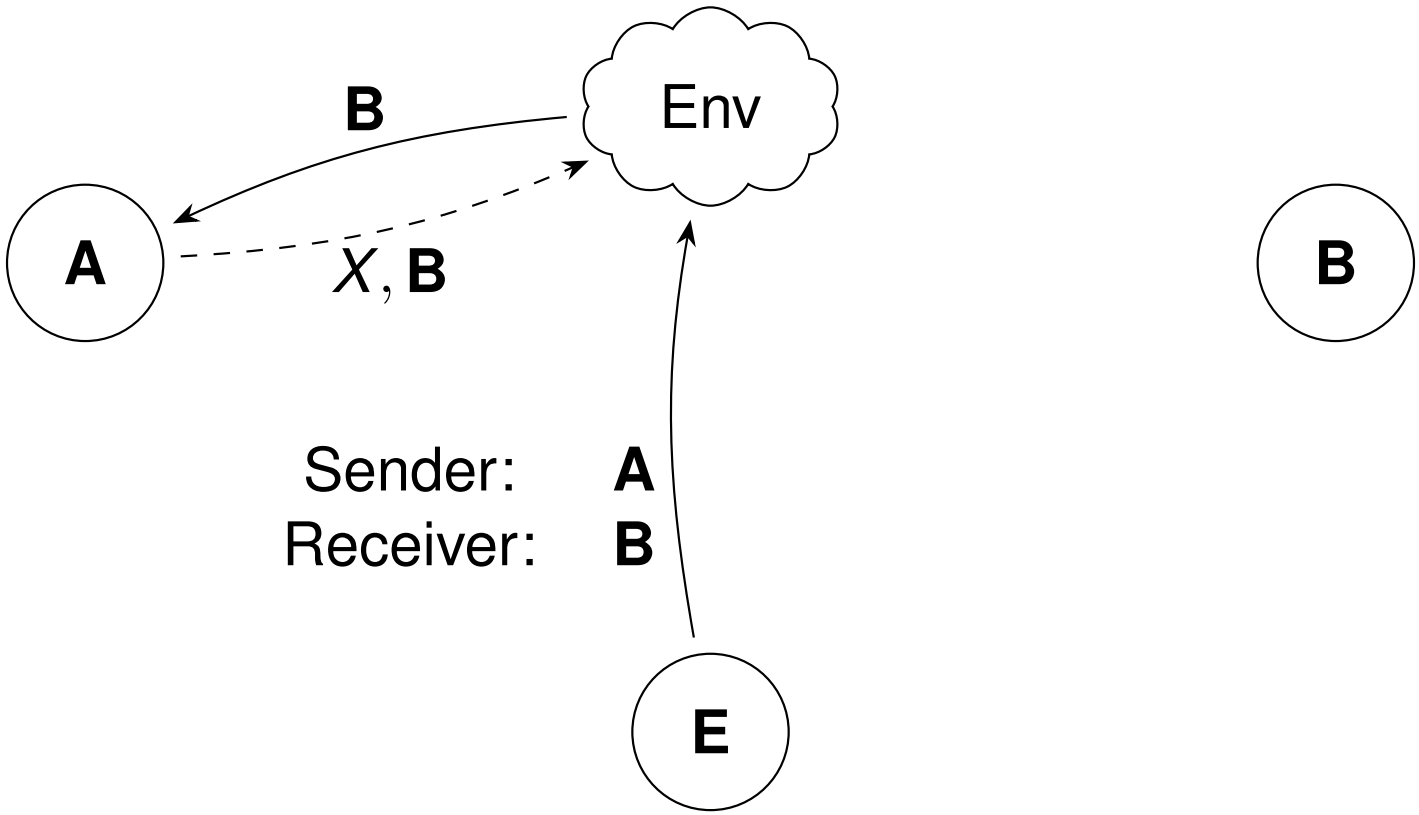

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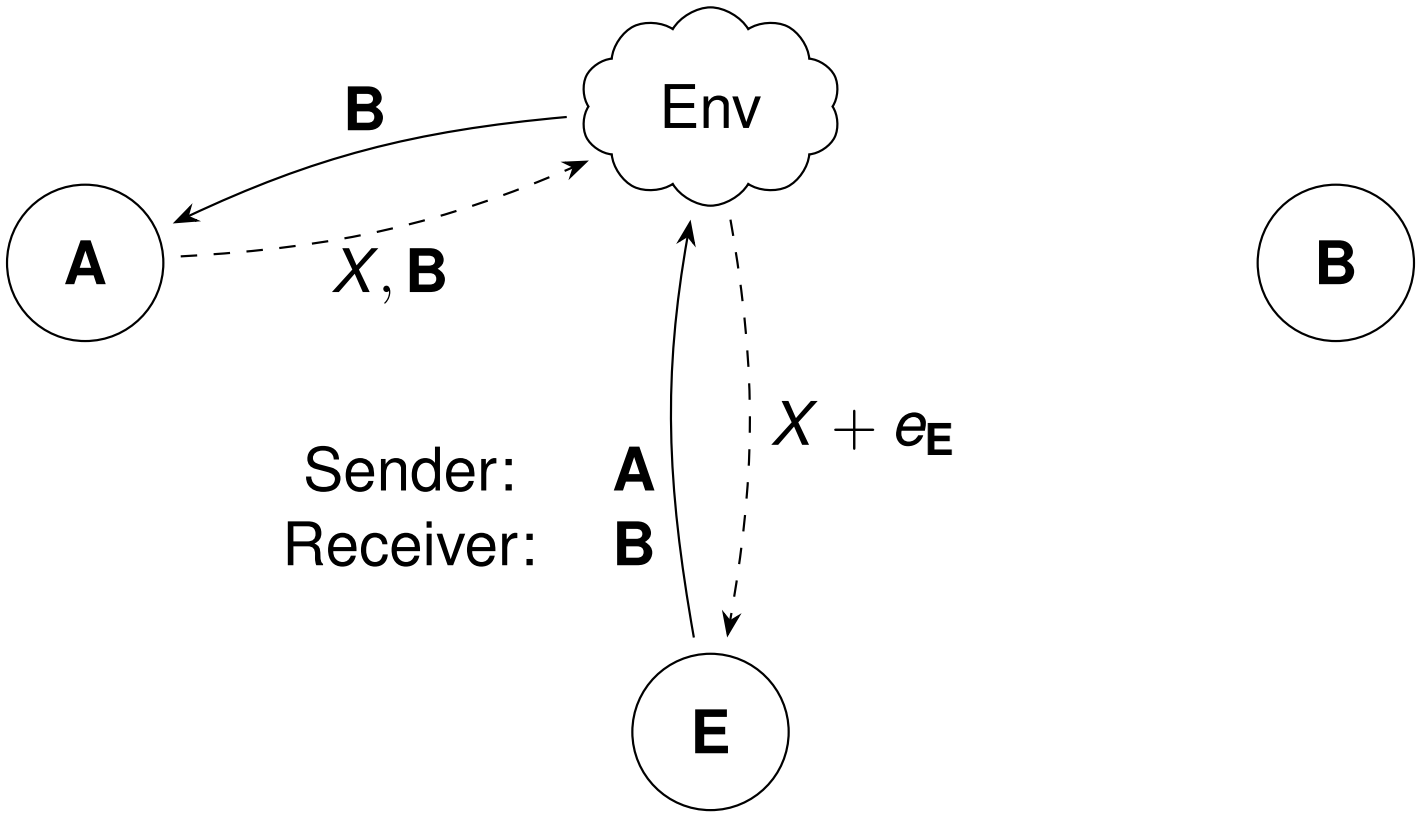

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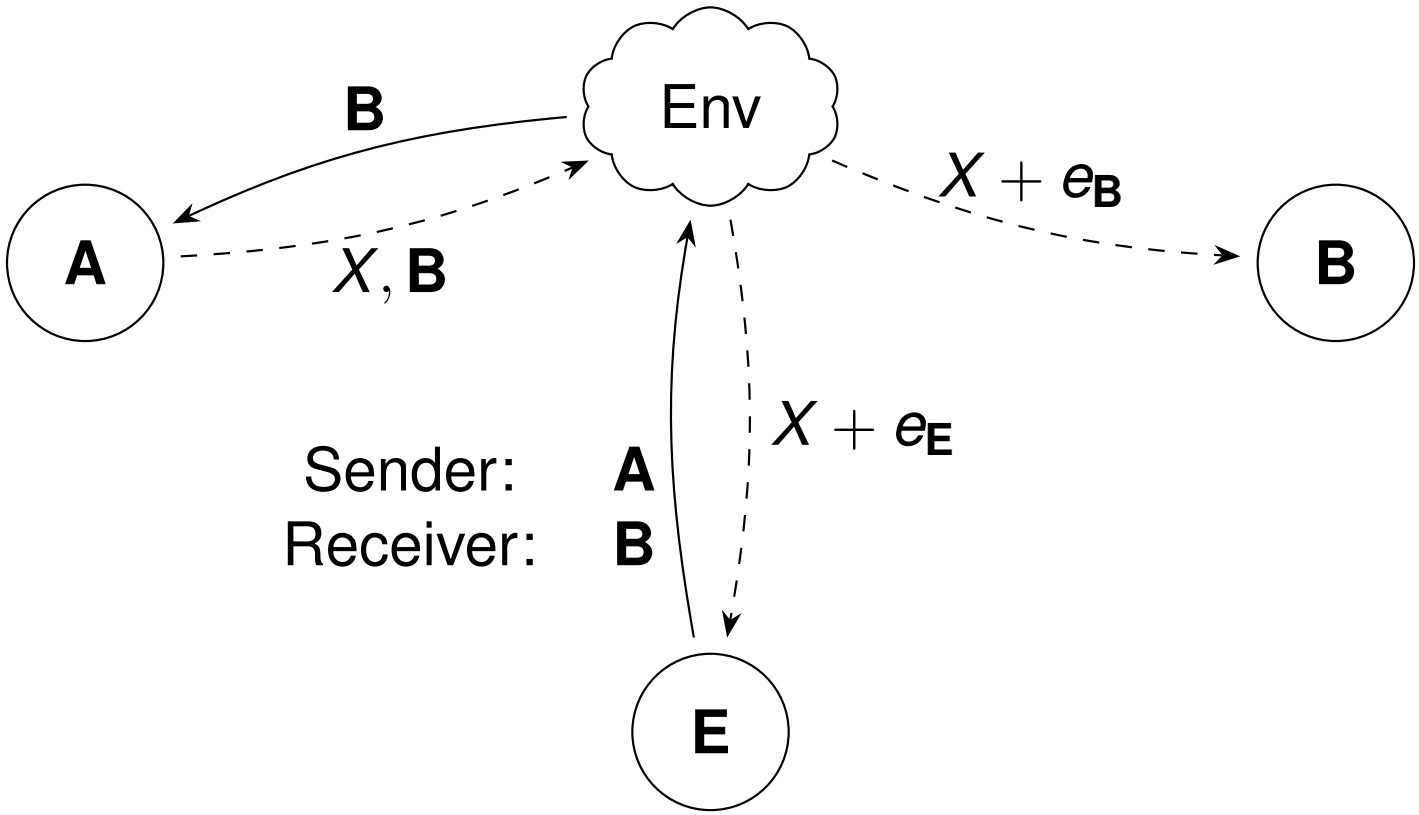

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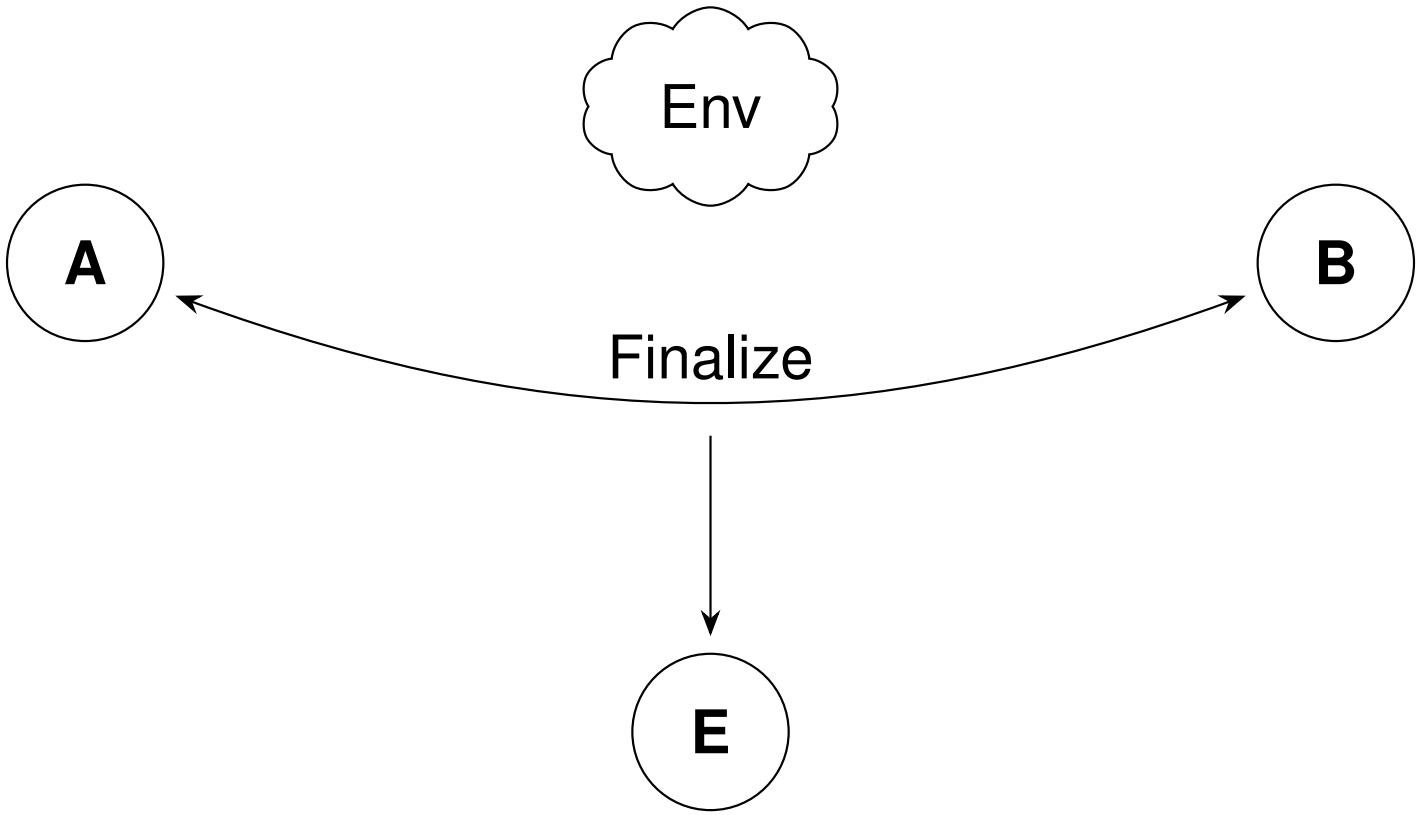

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- Since we want to use the keys, we have to be sure that they are secure.
- Use previous keys and be happy if keys are eventually secure.

Questions?