Error Resilience in the Number-Theoretic Transform

PQC acceleration for safety critical applications

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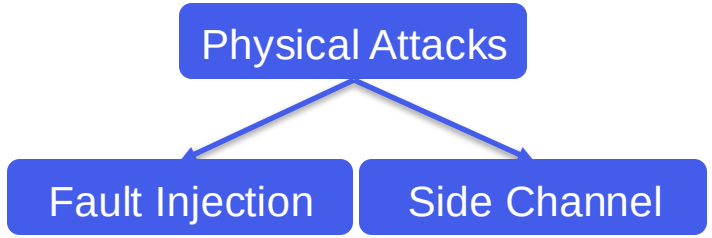
Motivation

Post-Quantum Cryptography: Algorithms based on **classical** operations that resist attacks from quantum computers

Hard problems:

- Lattice based
- Code based
- Hash based
- Multivariate based
- Isogeny based



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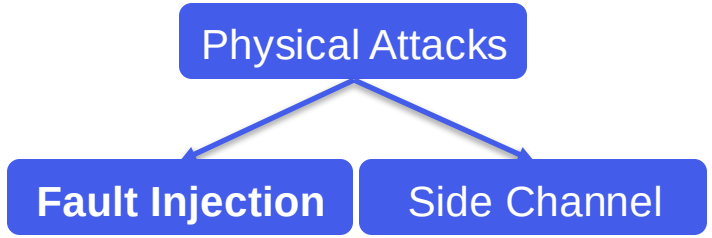
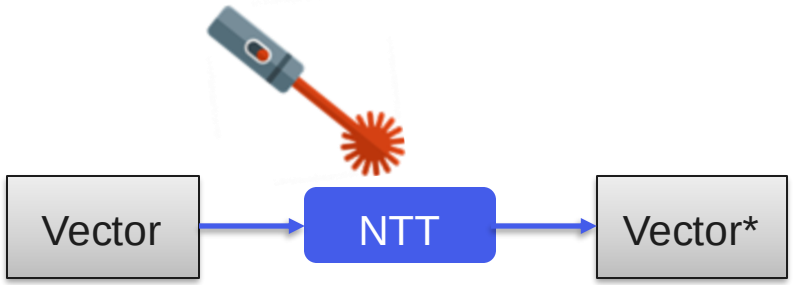
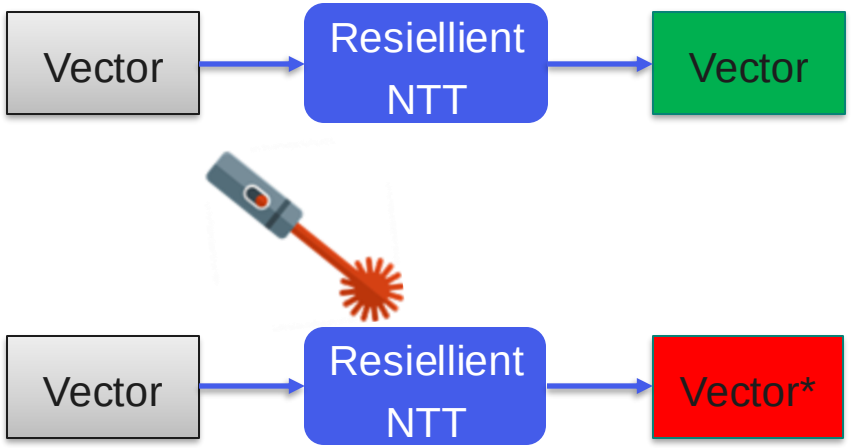


Example





Number Theoretic Transform

- **Module-Lattice-Based**
  - Use polynomial ring $R_q = \mathbb{Z}_q[X]/\phi(x)$ instead of $\mathbb{Z}_q$
  - Better efficiency
  - Same security

➡ Fast arithmetic on $R_q = \mathbb{Z}_q[X]/\phi(x)$ necessary

- Schoolbook-Multiplication” $O(n^2)$ operations
- Fast Multiplication $O(n \log(n))$ operations

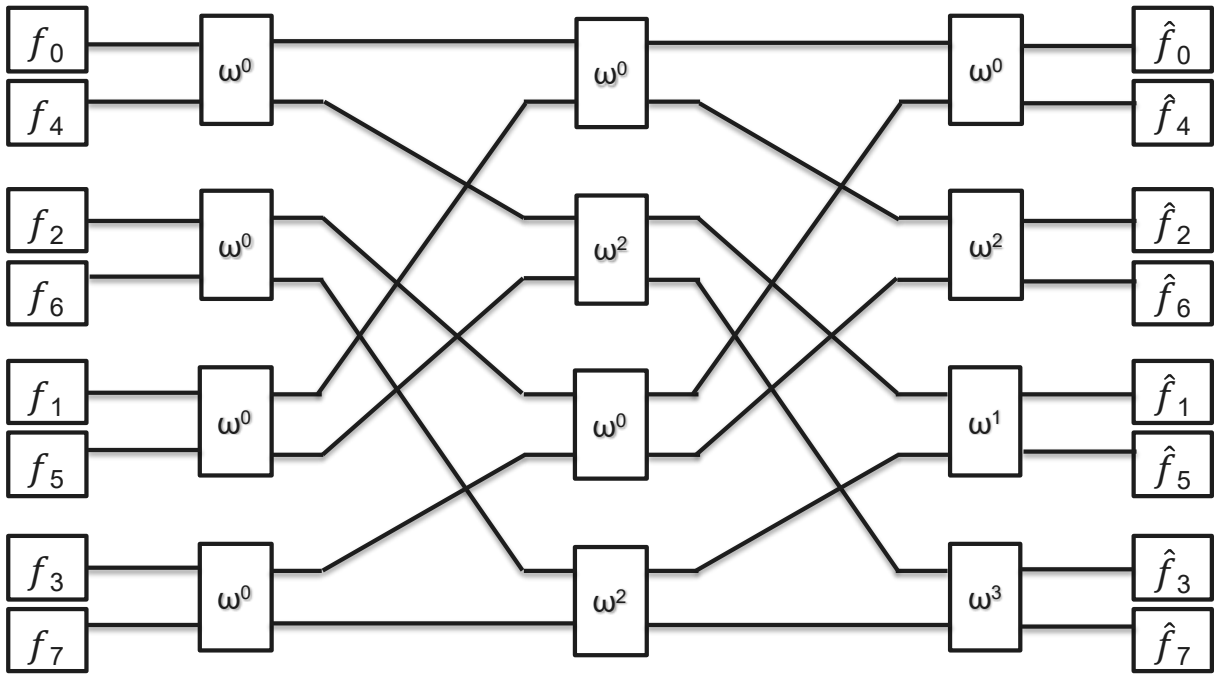
- **Fast Multiplication**

Calculate product as

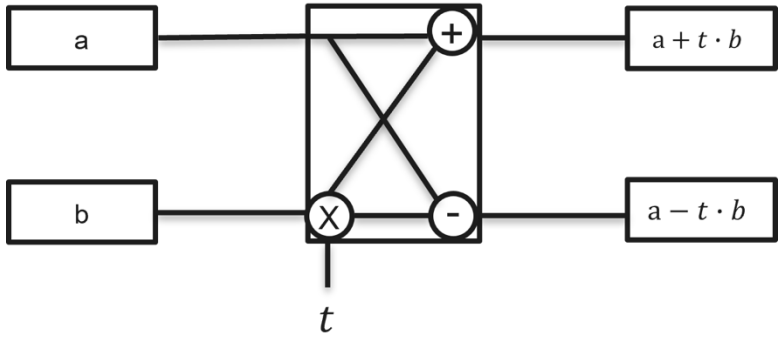
$$f \times g = NTT^{-1}\big(NTT(f) \circ NTT(g)\big)$$

where $\circ$ is the elementwise multiplication

Number Theoretic Transform

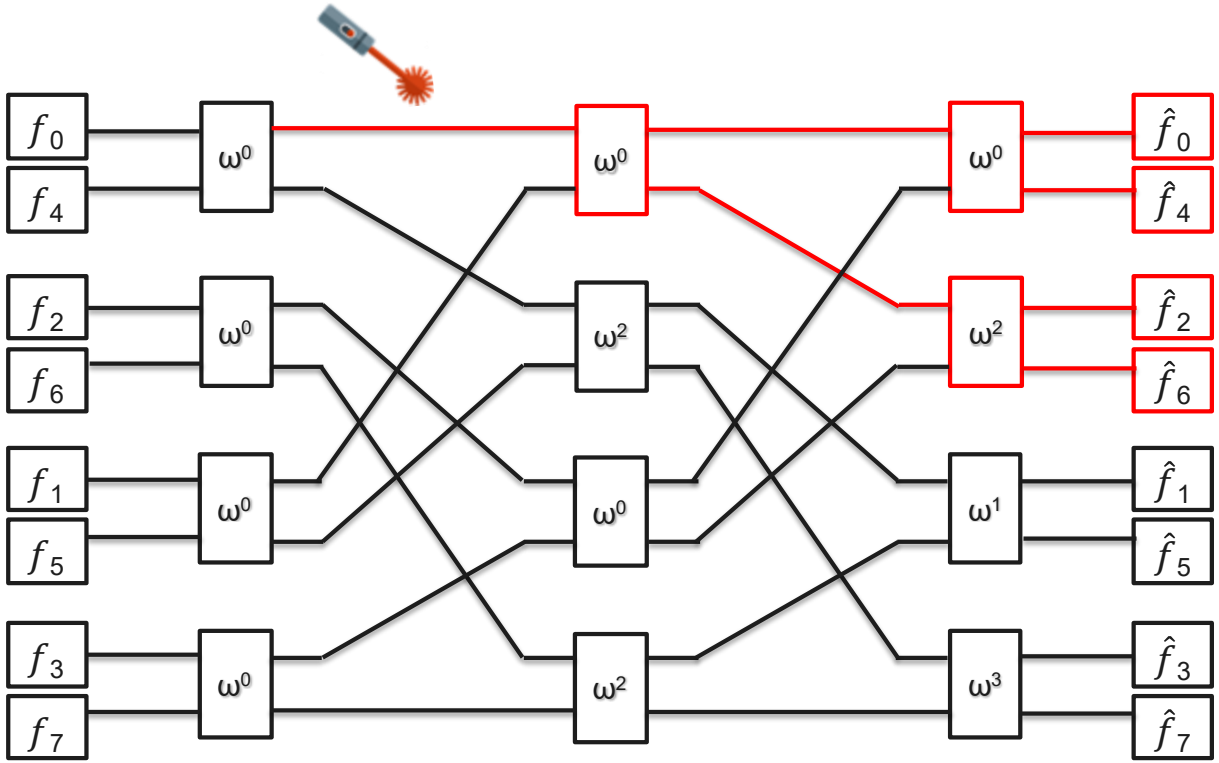


General structure of an NTT



Breakdown into core operations

Number Theoretic Transform



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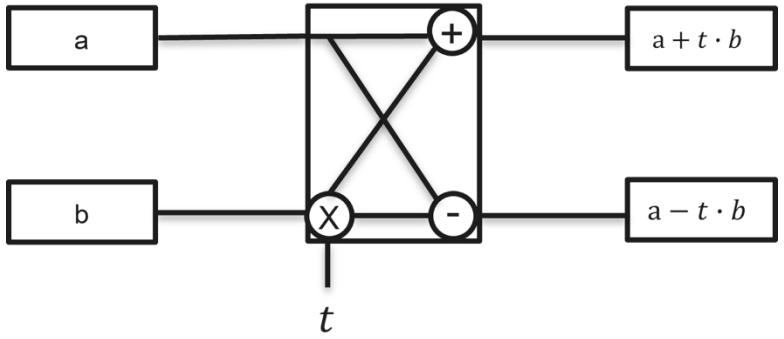
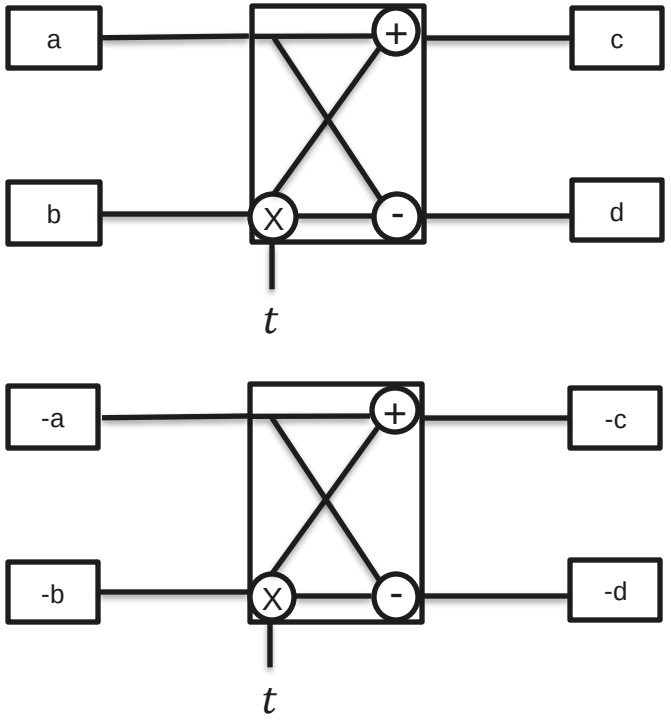


Breakdown into core operations

Recomputing (Sarker 2019)

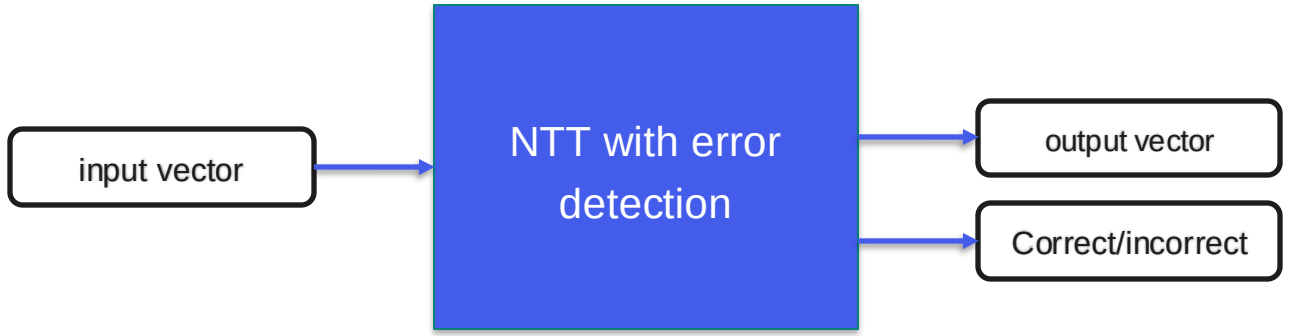
Techniques in the Literature

- Each butterfly operations is done twice → compare results



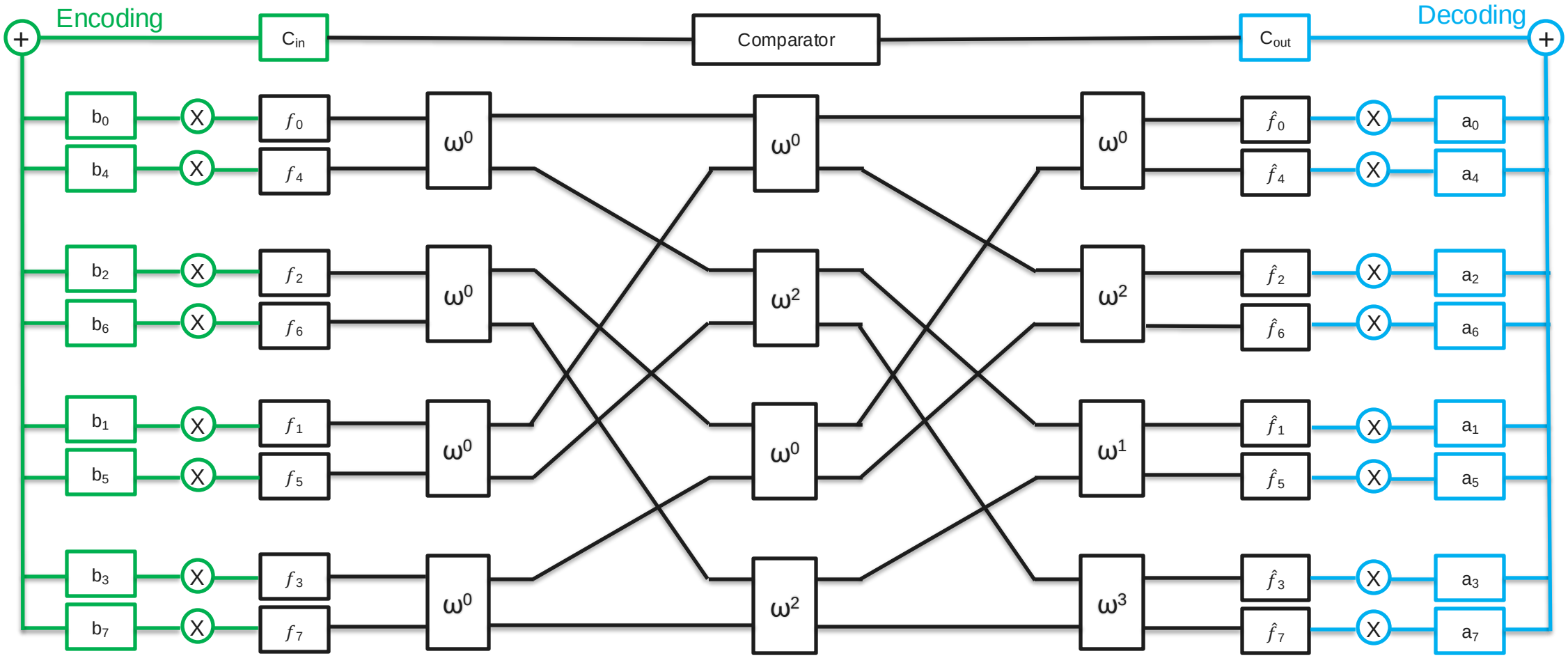
- Disadvantages:
  - high overhead
  - tests only butterfly calculations

- Our goal:
  - Test the whole NTT block with less overhead



Weighted Checksums (Tao, Hartmann, and Chen 1990 and Wang and Jha 1994)

Applying methods known for FFT



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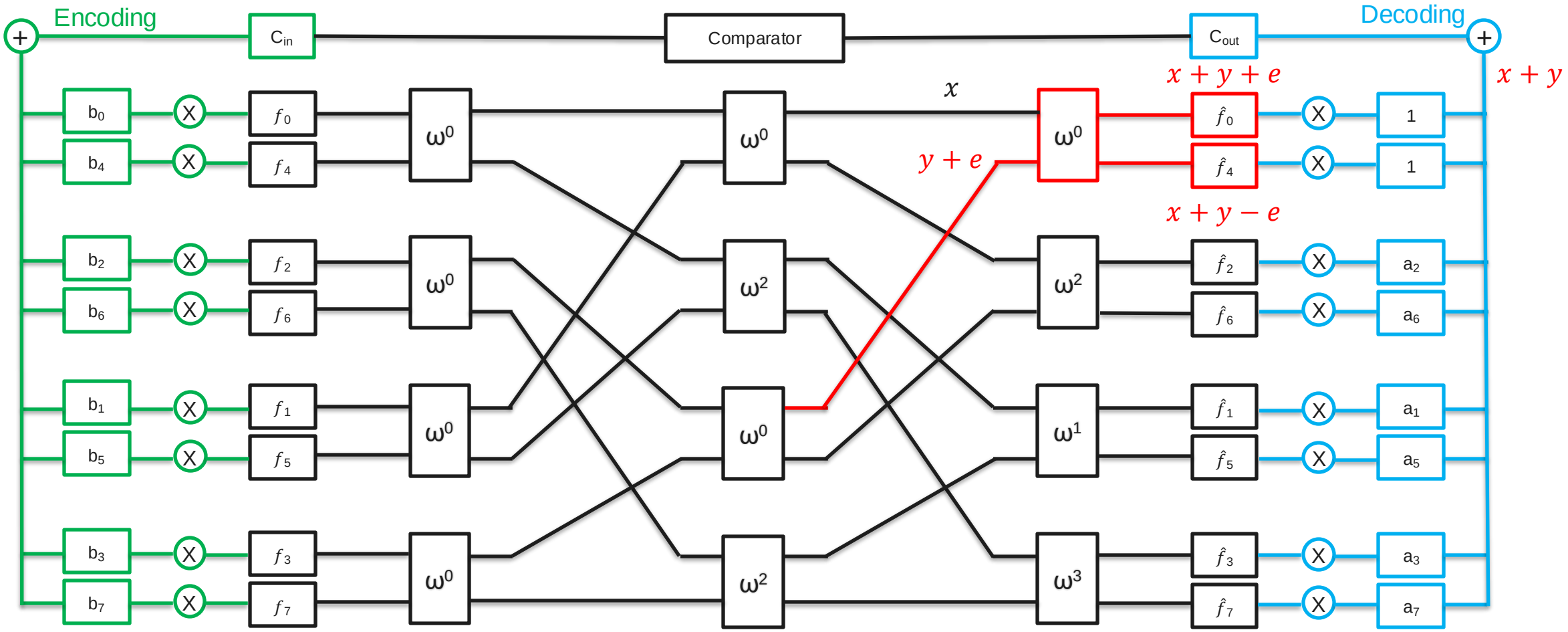


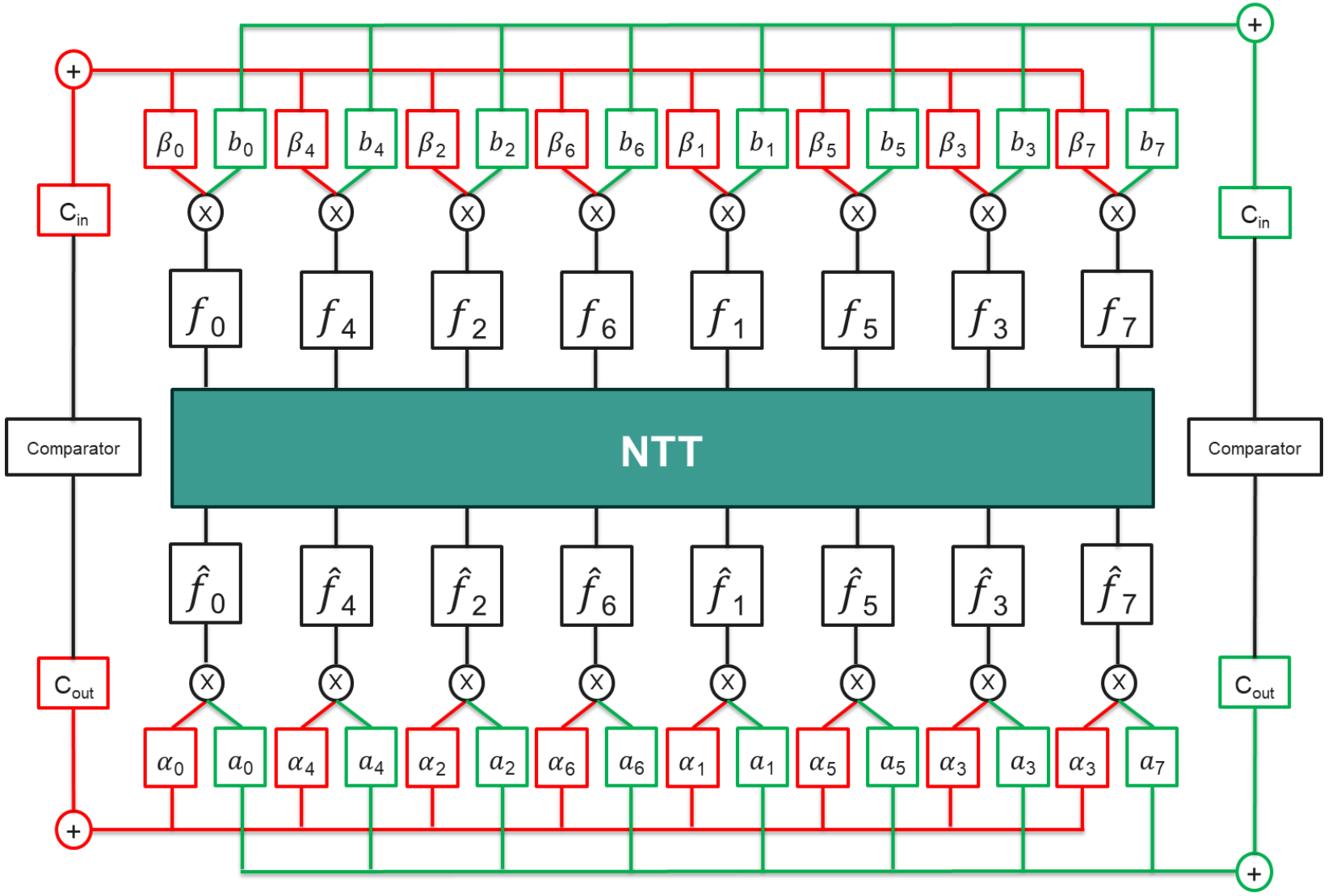
a_0 *and* a_4 *can not be the same*

Our work

- Derive conditions on **a's** to detect **one and two Faults in NTT network**
- One Fault:
  - show that **a** exists that meets these conditions for the parameter sets of **Dilithium, Falcon and Kyber**
  - we provide an efficient implementation that requires only $\boldsymbol{N}$ ***extra multiplications*** (recomputing $\boldsymbol{N/2\ log\ N}$ extra multiplications)

Scheme	Dilithium	Kyber	Falcon I	Falcon V
Overhead	0.25	0.286	0.222	0.2

Two error detection



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- Two Faults:
  - show that **a's** exist that meet these conditions for the parameter sets of **Dilithium**
  - upper bound: about $\boldsymbol{N^2}$restrictions on $\boldsymbol{a_i} \in \mathbb{Z}_q^N$

Scheme	Ring degree N	N^2	Modulus q
Dilithium	256	65536	$<$ 8380417
Kyber	256	65536	$>$ 3329
FALCON I	512	262144	$>$ 12289
FALCON V	1024	1048576	$>$ 12289

Future work

- Find **a's** that also work for Kyber and Falcon

- Error correction

- Find Fault injection attacks and countermeasures on other PQC schemes
  - 40 new NIST **Additional Digital Signature Schemes** in Round 1