THE NACA0012 AIRFOIL
PROJECT IN MA8502

The purpose of the project is to implement an incompressible Navier-Stokes solver and run a series of test-cases on the NACA0012 airfoil. The NACA airfoil is a particular 2D airfoil for which there exists an abundance of experimental windtunnel data and good numerical solvers.

In our case, we will not use any turbulence model and we will assume that the flow is incompressible. This will make our numerical solution not comparable to the ones found online. Thus, your results will be compared to mine and the other students in the class.

The equations. The equations to solve are

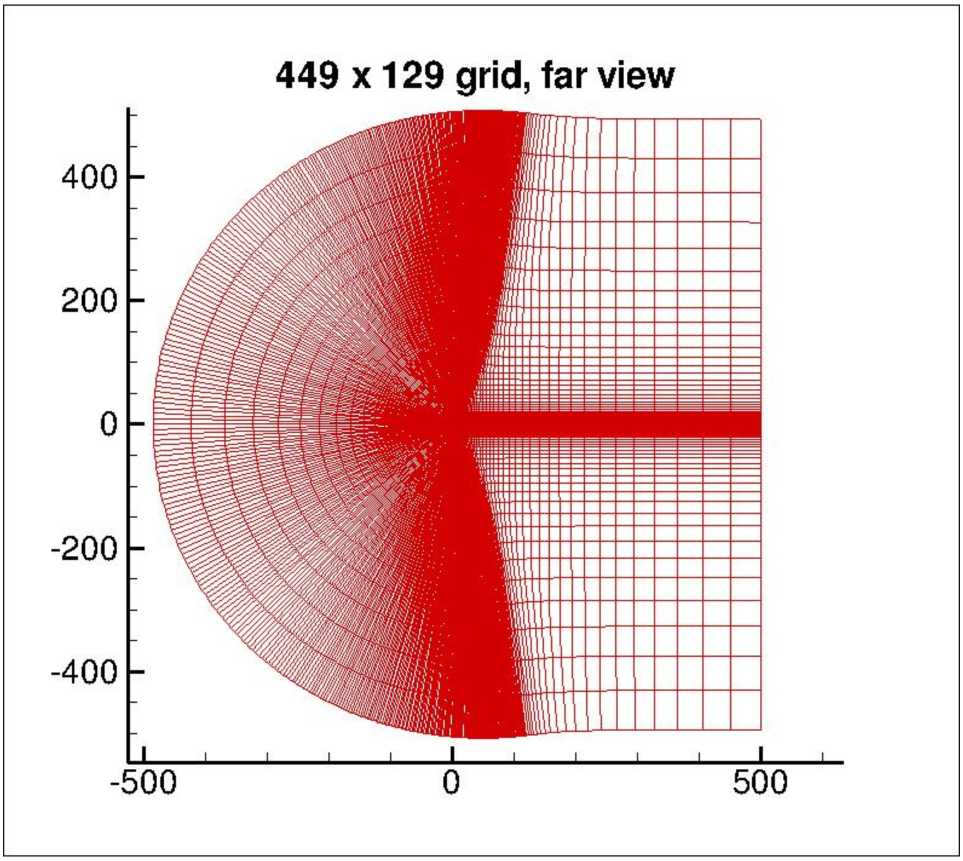
$$\begin{gathered} u_t + u \cdot \nabla u + \nabla p = \frac{1}{\mathrm{Re}} \Delta u + \bar{g}, \quad \text{in } (0,T) \times \Omega \\ \operatorname{div} u = 0, \quad \text{in } (0,T) \times \Omega, \end{gathered}$$

where $\bar{g} = [0, 0, -g]$, g is gravity constant, and Re is the Reynolds number.

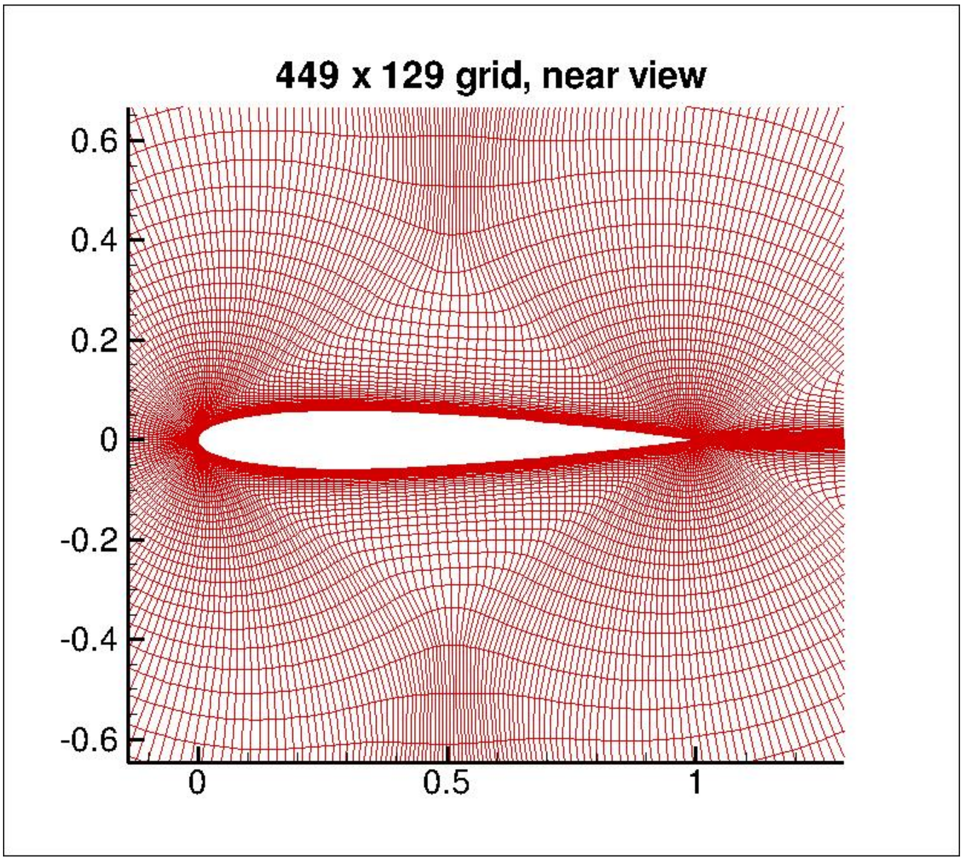
However, we shall assume that we have reached a stationary state, and instead solve the equations:

$$\begin{gathered} u \cdot \nabla u + \nabla p = \frac{1}{\mathrm{Re}} \Delta u + \bar{g}, \quad \text{in } \Omega \\ \operatorname{div} u = 0, \quad \text{in } \Omega, \end{gathered}$$

The domain $\Omega \subset \mathbb{R}^2$ is rather special and given by the grid:



Hence, the grid extends far outside of the airfoil. If one zooms-inn at the airfoil:



You can find the grid, in various resolutions at the course web-page.

The boundary conditions. On the semi-circular part, set the boundary condition

$$u(t,x) = \bar{U}\begin{pmatrix}\cos(\alpha)\\ \sin(\alpha)\end{pmatrix},$$

where α is the angle of attack of the airfoil and $\bar{U}$ is amplitude of the incoming wind (speed of the imagined aircraft).

On the airfoil, set no-slip boundary conditions ($u = 0$).

On the remaining parts of the boundary, set do-nothing conditions:

$$\left(\frac{1}{\mathrm{Re}}\left(\nabla u + \nabla u^T\right) - p\mathbb{I}\right)\cdot \nu = 0,$$

where ν is the outward unit normal to the boundary.

The task. Implement a numerical method that solve the stationary Navier-Stokes equations on the NACA0012 airfoil grid. Answer the following points:

- Let $\alpha = \frac{\pi}{20}$. Make a graph that shows how the lift and drag on the airfoil compare to the Reynolds number. That is, do the simulation for various values of Re and calculate the lift and drag on the airfoil.
- Let $\mathrm{Re} = 10^2$, calculate the lift and drag for various angle of attacks.
- Let $\mathrm{Re} = 10^6$, calculate the lift and the drag on the airfoil.

The Requirements. Here are the requirements:

- The code should be able to run on a standard desktop computer and give results within reasonable time.
- A first-order upwind method running on the coarsest grid is okey, but will not be awarded an A.
- The problem is to both achieve accuracy and speed.

Extra question for the super interested. Solve the non-stationary Navier-Stokes equations and set the inflow condition:

$$u(t,x) = \bar{U}\begin{pmatrix}\cos(\alpha(t))\\ \sin(\alpha(t))\end{pmatrix}, \qquad \alpha(t) = \frac{\pi}{4}|\sin(t)|.$$

Let $\mathrm{Re} = 10^6$ and plot how the lift and drag as functions of time.