Norwegian University of Science
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Department of Mathematical
Sciences

MA8404 Numerical solution of time dependent differential equations Autumn 2019

Exercise set 3

1 **a)** Which of the following linear multistep methods are convergent?
Find the order p and the error constant C_{p+1} for each method.

$$y_{n+2} + y_{n+1} - 2y_n = \frac{h}{4}\left[f(t_{n+2}, y_{n+2}) + 8f(t_{n+1}, y_{n+1}) + 3f(t_n, y_n)\right].$$

$$y_{n+3} + \frac{1}{4}y_{n+2} - \frac{1}{2}y_{n+1} - \frac{3}{4}y_n = \frac{h}{8}\left[19f(t_{n+2}, y_{n+2}) + 5f(t_n, y_n)\right].$$

$$y_{n+2} - y_{n+1} = \frac{h}{3}\left[3f(t_{n+1}, y_{n+1}) - 2f(t_n, y_n)\right].$$

b) Use the root locus curve idea to plot the stability regions of the convergent method(s).

2 Construct and analyse a variable stepsize BDF-2 method.

a) Given the points (t_n, y_n), (t_{n+1}, y_{n+1}) and (t_{n+2}, y_{n+2}) where $t_{n+1} = t_n + h$ and $t_{n+2} = t_{n+1} + \omega h$, meaning that the stepsize change from one step to the next by a factor of ω.

- Construct the interpolation polynomial $p(t)$ through these points.
- The BDF-method is given by $p'(t_{n+1}) = f(t_{n+2}, y_{n+2})$.
- Write the method in the form

$$y_{n+2} + \alpha_1(\omega)y_{n+1} + \alpha_0(\omega)y_n = h\beta(\omega)f_{n+2}.$$

b) Discuss the convergence properties of the method. Are there any restrictions on the choice of ω?

c) Investigate how the stability region of the method is affected by the choice of ω.