

## MA8403 Week 6

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### 1. Question 1

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Let  $C$  be a  $G$ -coefficient system.

1. Show that:

$$\mathrm{Hom}_{\mathrm{coeff}}(C, \mathbb{Z}) \cong \mathrm{Hom}_{\mathrm{AbGrps}}(C(G/e)/G, \mathbb{Z}).$$

Use this to deduce that for a  $G$ -space  $X$  we have

$$H_G^*(X; \mathbb{Z}) \cong H^*(X/G; \mathbb{Z}).$$

2. Show that:

$$\mathrm{Hom}_{\mathrm{coeff}}(C, \mathrm{Inf}_e^G(\mathbb{Z})) \cong \mathrm{Hom}_{\mathrm{AbGrps}}(C(G), \mathbb{Z})$$

Use this to deduce that for a  $G$ -space  $X$  we have

$$H_G^*(X; \mathbb{Z}) \cong H^*(X^G; \mathbb{Z}).$$

### 2. Question 2

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Let  $L$  be the  $C_p$ -coefficient system defined as the kernel  $L := \ker(\mathbb{F}_p \rightarrow \mathrm{Inf}_e^{C_p}(\mathbb{F}_p))$ ; in other words,  $L(C_p/C_p) = 0$  and  $L(C_p/e) = \mathbb{F}_p$ . Use the results in Question (1) to show that for a  $G$ -space  $X$  we have

$$H_{C_p}^n(X; L) \cong H^n(X/C_p, X^{C_p}; \mathbb{F}_p) \cong \tilde{H}^n((X/X^{C_p})/C_p; \mathbb{F}_p)$$