MA8403 Week 2

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1. Question 1

Consider a C_2-Mackey of the form

$$\begin{tikzcd} \mathbb{Z} \arrow[d, bend right] \\ \mathbb{Z}_{sgn} \arrow[u, bend right] \arrow[loop below] \end{tikzcd}$$

where $\mathbb{Z}_{sgn}$ denotes $\mathbb{Z}$ with the C_2 action that sends 1 to -1. Show that this is necessarily a "split" Mackey functor, in the sense that both the restriction and transfer maps are necessarily zero.

2. Question 2

Let $\underline{M}$ be a C_2-Mackey functor. Describe the induced Mackey functor $\mathrm{Ind}_{C_2}^{C_4} \underline{M}$.

3. Question 3

Recall that the Burnside ring for $G = C_2$ is $A(C_2) \cong \mathbb{Z}\{C_2/C_2, C_2/e\}$ and that the Burnside Mackey functor is given by

$$\mathbb{A}_{C_2} = \begin{tikzcd} \mathbb{Z}\{C_2/C_2, C_2/e\} \arrow[d, bend right, "(1\ 2)"'] \\ \mathbb{Z}\{e\} \arrow[u, bend right, "\binom{0}{1}"'] \arrow[loop below] \end{tikzcd}$$

1. Show that, as a ring, $A(C_2) \cong \mathbb{Z}[a_1]/(a_1^2 = 2a_1)$, where a_1 corresponds to C_2/e.
   Hint: In G-sets there is a Mackey decomposition,
   $$G/H \times G/K \cong \coprod_{HgK \in H \backslash G/K} G/(H^g \cap K).$$
2. Show that the linearization homomorphism $\mathbb{A}_{C_2} \to RO(C_2)$ is an isomorphism (this is special to $G = C_2$).