

MA 8205

Exercises 1 -

for Friday 7.2

# Exercises.

① Find all indec. representations of

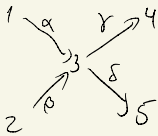
$$K \left( \begin{array}{ccc} 1 & \xrightarrow{\alpha} & 2 \\ & \nwarrow \beta & \nearrow \gamma \\ & 3 & \end{array} \right) / \langle \alpha\beta\gamma \rangle$$

Determine which are projectives (injectives).

Find projective resolutions of all simples.

②

Let  $Q$



a) Show that  $\mathbb{C}Q$  has an infinite number of indec. representations with composition factors  $S_1, S_2, S_3, S_3, S_4, S_5$ .

$$\left( \begin{array}{ccccc} & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{array} \right)$$

b) Let  $I = \langle \alpha\gamma, \beta\delta \rangle$

Find the projectives and injectives.

Compute projective resolutions of simples.

③  $\Lambda$  Nakayama algebra

Assume  $ll(\Lambda) = ll(P)$  for an indec. projective  $P$ .

Show that  $P$  is injective.

# Suggested exercises from the lectures.

- (4) (From RT-2):  $0 \rightarrow A \rightarrow B \rightarrow C \rightarrow 0$  exact seq. of right modules over any ring. Show that  $A, C$  fin-gen  $\Rightarrow B$  fin-gen.

(RT-2)

- (5) Show that  $\begin{bmatrix} K & 0 & 0 & 0 \\ 0 & K & 0 & 0 \\ K & K & K & 0 \\ K & K & K & K \end{bmatrix}$  is an algebra ( $K = \text{field}$ )

and find the corresponding algebra  $KQ/I$ .

(RT-5)

- (6)  $D \text{Hom}_A(-, A) \simeq - \otimes_A D A$  natural equivalence (isomorphism of functors)

Hint: <sup>use</sup> adjunction to prove  $\text{Hom}_A(-, A) \simeq D(- \otimes_A A)$ , then use  $DA \simeq \text{id}$ .

(RT-5)

- (7) <sup>mod. over some algebra  $KQ$</sup>  Find an  $A$ -module  $M$  such that  $\text{top } M$  is simple and  $\text{soc } M \supset \text{simple}$ , but  $M$  is not uniserial.

(8)

Prove

Let  $\ell(M) = n$  and  $M = M_0 \supseteq M_1 \supseteq \dots \supseteq M_n = (0)$

a composition series.

$\ell(M) - 1$

$$\bigoplus_{i=0}^{\ell(M)-1} M_i / M_{i+1} \simeq \bigoplus_{i=0}^{n-1} M_i / M_{i+1}$$