MA 8205

Exercises 1 –

for Friday 7.2

Exercises

① Find all indec. representations of

$$K\left(\begin{tikzcd} 1 \arrow[r, "\alpha"] & 2 \arrow[dl, "\beta"] \\ 3 \arrow[u, "\gamma"] & \end{tikzcd}\right) \Big/ \langle \alpha\beta\gamma \rangle .$$

Determine which are projectives (injectives).

Find projective resolutions of all simples.

② Let Q

$$\begin{tikzcd} 1 \arrow[dr, "\alpha"] & & 4 \\ & 3 \arrow[ur, "\gamma"] \arrow[dr, "\delta"] & \\ 2 \arrow[ur, "\beta"] & & 5 \end{tikzcd}$$

a) Show that $\mathbb{C}Q$ has an infinite number of indec. representations with composition factors $S_1, S_2, S_3, S_3, S_4, S_5$.

$$\left(\begin{tikzcd} k \arrow[dr] & & k \\ & k^2 \arrow[ur] \arrow[dr] & \\ k \arrow[ur] & & k \end{tikzcd}\right)$$

b) Let $I = \langle \alpha\gamma, \beta\delta \rangle$

Find the projectives and injectives.

Compute projective resolutions of simples.

③ Λ Nakayama algebra

Assume $ll(\Lambda) = ll(P)$ for an indec. projective P.

Show that P is injective.

Suggested exercises from the lectures.

(4) (From RT-2): $0 \to A \to B \to C \to 0$ exact seq. of right modules over any ring. Show that A, C fin.-gen $\Rightarrow B$ fin.-gen.

(5) (RT-2) Show that
$$\begin{bmatrix} k & 0 & 0 & 0 \\ 0 & k & 0 & 0 \\ k & k & k & 0 \\ k & k & k & k \end{bmatrix}$$
is an algebra (k = field)

and find the corresponding algebra kQ/I.

(6) (RT-5) $D\,\mathrm{Hom}_\Lambda(-,\Lambda) \simeq - \otimes_\Lambda D\Lambda$ natural equivalence (isomorphism of functors)

Hint: use adjunction to prove $\mathrm{Hom}_\Lambda(-,\Lambda) \simeq D(-\otimes D\Lambda)$, then use $DD \simeq \mathrm{id}$.

(7) (RT-5) Find an indec. module M over some algebra kQ such that $\mathrm{top}\,M$ is simple and $\mathrm{soc}\,M$ is simple, but M is not uniserial.

(8) Prove. Let $l(M) = n$ and $M = M_0 \supseteq M_1 \supseteq \dots \supseteq M_n = (0)$ a composition series.

$$\bigoplus_{i=0}^{ll(M)-1} M\underline{r}^i / M\underline{r}^{i+1} \simeq \bigoplus_{i=0}^{n-1} M_i / M_{i+1}$$