EXAM PREPARATION FOR MA8203: ALGEBRAIC GEOMETRY

UPDATED LAST: MAY 7, 2021

Update 7 May: Chapter VIII from Perrin will not be on the exam. I slightly shortened the list of theorems to know. I have also updated the stars on the exercise sheet.

The exam will cover all content covered in class and assigned work (reading and exercises). This includes:

- Chapters I – VII from Perrin
- Portions from Hartshorne book (chapter II, sections 1–5)

In order to help focus your studying, here are some more specifics. As a caveat, these lists are not intended to be complete or all-encompassing.

(1) You should be able to provide a careful proof for these results from class:
   - Theorem I.3.2: correspondence between irreducible affine algebraic sets and prime ideals
   - Hilbert's Nullstellensatz (Theorem I.4.3). You may assume the weak nullstellensatz (I.4.1) proven in MA8202.
   - Hartshorne ch. II, section 1, Proposition 1.1 (proven in class 16 Feb): morphism of sheaves is an isomorphism if and only if it is isomorphism on all stalks.
   - Proposition IV.1.7: relationship between dimension of affine algebraic variety and Krull dimension.
   - Lemmas VI.2.3, VI.2.4, and VI.2.6: components of proof for Bézout's theorem
   - Proposition VII.2.3 and Theorem VII.2.9: global sections as Cech cohomology

(2) Here is a list of topics and results you should be very familiar with (and prepared to discuss examples, big ideas, portions of the proofs, etc.). Again, this list is not intended to be comprehensive.
   - Affine algebraic sets; projective algebraic sets; Zariski topology; affine and projective $V(-)$ and $I(-)$; irreducibility; affine and projective Nullstellensatz; correspondence between geometry and algebra (prime ideals, max ideals, etc.); projective space (especially $\mathbb{P}^2$ and $\mathbb{P}^1$); sheaves (including examples and basic properties); structural sheaves; (affine and projective) algebraic varieties; stalks and local rings; dimension of algebraic varieties (both topological and algebraic); tangent spaces; singular points; relationship between regular local rings and smoothness; intersection multiplicities; results and examples pertaining to Bézout's theorem; construction of Cech complex and Cech corresponding cohomology; basics of schemes

(3) Finally, I have added a blue star ($\bigstar$) for those exercises which you should be able to do for the exam. (See the exercise PDF list.)

(4) Don't forget that the first 10 minutes of the exam is for you to present a prepared problem/exercise/paper of your choosing. If you have questions about what is appropriate for this, just ask.