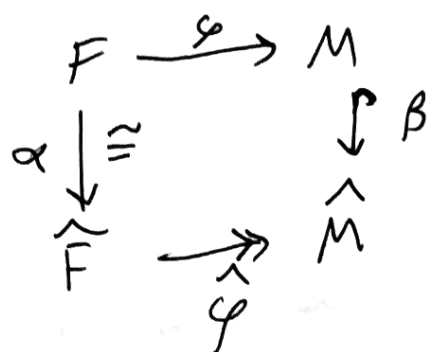


We then have:



- α is iso as F is free A -mod and $A \cong \hat{A}$.
 - As $\bigcap_{n \geq 1} M_n = 0$, we get β injective.
 - surj of $\hat{F} \rightarrow \hat{M} \rightarrow \beta$, and hence $\beta \circ \hat{\varphi}$, are surjective.
- Thus x_1, \dots, x_r generate M as A -module. \square

Cor 10.25

In the setup of 10.24, if $G(M)$ is Noetherian,
~~then~~ as a $G(A)$ -mod, then M is Noeth. A -mod.

Pf: Let $M' \subseteq M$ be submod.
 Let $M'_n = M' \cap M_n$, so (M'_n) is α -filt. of M'
 and $M'_n / M'_{n+1} \hookrightarrow M_n / M_{n+1}$ inj hence
 we have an embedding of $G(M')$ into $G(M)$.

As $G(M)$ Noeth, $G(M')$ is f.g.
 Also, ~~$\bigcap M'_n = 0$~~ $\bigcap M'_n \subseteq \bigcap M_n = 0$
~~So~~ So (10.24) $\Rightarrow M'$ is f.g. A -mod
 $\Rightarrow M$ is Noeth A -mod. \square

Theorem 10.26

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If A is Noetherian ring, $\mathfrak{a} \subseteq A$,
then \mathfrak{a} -adic completion \hat{A} of A is Noeth.

PF:
By (10.22), $G_{\mathfrak{a}}(A)$ is Noetherian and
 $G_{\mathfrak{a}}(A) \cong G_{\hat{\mathfrak{a}}}(\hat{A})$ is Noetherian.
By (10.25), taking $M = \hat{A}$ (filtered by $\hat{\mathfrak{a}}^n$ hence $\bigcap \hat{\mathfrak{a}}^n = 0$)
we get \hat{A} is Noetherian (as a module over itself hence as a ring). \square

Cor 10.27:

If A is Noetherian ring, then

$A[x_1, \dots, x_n]$ is Noetherian.

PF: $A[x_1, \dots, x_n]$ is Noetherian by HBT,
and $A[x_1, \dots, x_n]$ is the completion
of $A[x_1, \dots, x_n]$ for the (x_1, \dots, x_n) -adic
topology (exercise). \square