EXERCISES FOR MA8107 - FALL 2014

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(1) Let $C^n(\mathbb{T})$ be the space of n-times continuously differentiable functions on the circle $\mathbb{T}$. Show that $C^n(\mathbb{T})$ is a Banach algebra with respect to pointwise multiplication for the following norm:

$$\|f\|_{C^n} := \sum_{j=0}^{n} \frac{\|f^{(j)}\|_\infty}{j!}.$$

(2) Let v be a submultiplicative weight function on the integers $\mathbb{Z}$. Show that there exists a constant $a \geq 0$ such that $v(k) \leq e^{a|k|}$ for all $k \in \mathbb{Z}$.

(3) Show the following: If f is in $C^1(\mathbb{T})$, then f is in the Wiener algebra $\mathcal{W}$ and

$$\|f\|_{\mathcal{W}} \leq \|f\|_\infty + \frac{\pi^2}{3} \sum_{k \neq 0} k^2 |\widehat{f}(k)|^2.$$

Hint: Use that $\widehat{f'}(k) = 2\pi i k \widehat{f}(k)$ and Parseval's Theorem for Fourier series.

(4) Let $\mathcal{A}$ be a Banach algebra and we denote by $\sigma(a)$ the spectrum of an element a in $\mathcal{A}$. Then we have that

$$\sigma(ab) \cup \{0\} = \sigma(ba) \cup \{0\}.$$

(5) Let $a = (a_n)_{n \geq 0}$ be a sequence in $\ell^2(\mathbb{N})$. Then we define the shift operator $Sa = (a_{n+1})$ on $\ell^2(\mathbb{N})$. Determine S^* and compute the spectrum of SS^* and S^*S.