MA8107 OPERATOR ALGEBRA
EXERCISES 2

EXERCISE 1: Let X be a locally compact Hausdorff space. Show that there is a bijective correspondence which preserves inclusion between the set of open subsets of X and closed ideals of $C_0(X)$. (Hint: see page 16 in [de la Harpe and Jones]).

EXERCISE 2: Let A be a unital C^*-algebra. Remember that an element p of A is called a projection if $p = p^* = p^2$, and that an element u of A is called a unitary if $u^*u = uu^* = 1$.

1. Let p be a projection. Show that $\sigma(p) \subseteq \{0,1\}$.
2. Suppose that p is normal and that $\sigma(p) \subseteq \{0,1\}$. Show that p is a projection.
3. Let u be a unitary. Show that $\sigma(u) \subseteq \{\lambda \in \mathbb{C} \mid |\lambda| < 1\}$.
4. Suppose that u is normal and that $\sigma(u) \subseteq \{\lambda \in \mathbb{C} \mid |\lambda| < 1\}$. Show that u is a unitary.
5. Show that if $a \in A$ is selfadjoint (i.e., $a^* = a$), then e^{ia} is a unitary.
6. Show that if u is unitary and $\sigma(u) \neq \{\lambda \in \mathbb{C} \mid |\lambda| < 1\}$, then there exists a selfadjoint element $a \in A$ such that $e^{ia} = u$.

EXERCISE 3: Exercise 7 on page 74 of [Murphy].

EXERCISE 4: Exercise 8 on page 74 of [Murphy].

EXERCISE 5: Let X be a compact Hausdorff space. For each $x \in X$ let δ_x be the map from $C(X)$ to $\mathbb{C}$ given by $\delta_x(f) = f(x)$. Show that the map

$$x \mapsto \delta_x$$

is a homeomorphism from X to $\Omega(C(X))$. (Hint: see Theorem 2.1.15 of Murphy).