COHEN-MACAULAY MODULES IN REPRESENTATION THEORY

1. Talks

Contact Sondre Kvamme at sondre.kvamme@ntnu.no or Laertis Vaso at laertis.vaso@ntnu.no for the references or questions.

Talk 1 **Commutative Algebra I** Here basic definitions needed in commutative algebra are recalled. First define and discuss discrete valuation rings and Dedekind domains, following [1, Chapter 9]. Then discuss regular local rings, see [1, Chapter 11].

Talk 2 **Commutative Algebra II** Define and discuss the $\mathfrak{a}$-adic completion of a commutative ring R by an ideal $\mathfrak{a}$, following [1, Chapter 10]. In particular, state [1, Proposition 10.16 and Theorem 10.26], and discuss the examples of p-adic integers and the ring of formal power series.

Talk 3 **The Krull-Schmidt property for complete local rings**
Discuss the Krull-Schmidt property, following [10, Sections 1.1 and 1.2] and [3, Section 6B] (note that the proof of [10, Corollary 1.9] is not correct). In particular, explain that a module finite algebra over a complete noetherian local ring satisfies the Krull–Remak–Schmidt property.

Talk 4 **Auslander–Reiten sequences for orders over complete noetherian local rings**
Following [8], define orders, lattices, and isolated singularities (Note that lattices are sometimes called Maximal Cohen-Macaulay modules). Then discuss Auslander–Reiten sequences and Auslander–Reiten quivers [8, Sections 2.2 and 2.3]

Talk 5 **Maximal and hereditary orders**
Explain the basic properties of maximal and hereditary orders, including the necessary prerequisites, following [3, Section 26A], see also [8, Subsection 1.3]. In particular, state the existence result for maximal orders. Mention also [3, Theorem 26.12] on the relationship between maximal and hereditary orders (proof not necessary).

Talk 6 **Structure theory for maximal and hereditary orders**
Discuss the structure theorems for maximal and hereditary orders following [3, Section 26C], see also [8, Subsection 1.3].

Talk 7 **Bäckström orders**

Date: August 28, 2023.

Given the characterization of representation finiteness of Cohen-Macaulay modules over Bäckström orders, following [11]. In particular, state Dlab-Ringel's characterization of representation finiteness of valued graphs [11, Theorem 2.7].

Talk 8 **Ribbon graph orders**

Define Ribbon graph orders, following [5] and explain the proof why they are Bäckström orders [5, Section 1.3].

Talk 9 **Auslander–Reiten species for Bäckström orders**

Define subhereditary orders, generalized Bäckström orders, Auslander–Reiten species, and explain how to recover the Auslander–Reiten species for generalized Bäckström orders, following [12].

Talk 10 **Tiled orders** Define tiled orders over discrete valuation rings R, and give a characterization of representation finiteness, see [13, Section 7.2]. For $R = K[[t]]$ the Laurent polynomial ring, discuss tame orders, and give a characterization of when a tiled order is tame polynomial growth following [13, Section 7.2].

Talk 11 **Commutative algebra III**

Give basic definition in commutative algebra, following the first 2 pages in [9, Section 3.1] (e.g. depth, Maximal Cohen-Macaulay modules, regular rings, Gorenstein rings, ...). Also discuss Auslander-Buchsbaum formula (see [2, Theorem 1.3.3]) and Serre's characterization of regular rings [2, Theorem 2.2.7]. If time permits, introduce canonical modules and state Grothendieck's local duality theorem [4, Theorem 21.21].

Talk 12 **Commutative rings of finite type and artinian pairs**

This talk is based on [10, Chapter 3]. First explain the characterization of a commutative noetherian ring having finitely many indecomposables [10, Theorem 3.3]. Then discuss Artinian pairs and explain a necessary condition for an Artinian pair to be of infinite representation type [10, Theorem 3.7].

Talk 13 **Commutative rings of dimension 1 of finite Cohen-Macaulay type**

This talk is based on [10, Chapter 4]. Dicsuss the necessary and sufficient criterion for a local ring of dimension 1 to having finitely many indecomposables Cohen-Macaulay modules. Also mention [10, Theorem 4.13] by Greuel and Knörrer on ADE singularities.

2. Optional/Next Term

(i) **Bass orders** Explain the classification of Bass orders following [6, 7].

References

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