

## MA3408 Week 8

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### Question 1.

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Use the Serre spectral sequence to prove the following:

If  $F \rightarrow E \rightarrow S^n$  with  $n \geq 2$  is a fibration, then there is an exact sequence

$$\cdots \rightarrow H_i(F) \rightarrow H_i(E) \rightarrow H_{i-n}(F) \rightarrow H_{i-1}(F) \rightarrow H_{i-1}(E) \rightarrow \cdots$$

### Question 2.

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Let  $\pi: E \rightarrow B$  be a fibration with fiber  $F$ , let  $k$  be a field, and suppose  $\pi_1(B) = 0$ . Assume that the Euler characteristics  $\chi(B), \chi(F)$  are defined over the field  $k$ .<sup>1</sup> Then  $\chi(E)$  is defined, and

$$\chi(E) = \chi(B) \cdot \chi(F).$$

**Hint:** Construct an ‘Euler characteristic’ for the  $E_r$ -page of the Serre spectral sequence.

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<sup>1</sup>For a chain complex  $C$ , the Euler characteristic is the alternating sum of the ranks of the homology of the chain complex, assuming these ranks are all finite.