MA3408 Week 7

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February 23, 2022

Question 1.

Show that there is a fibration $S^2 \to \mathbb{C}P^3 \to S^4$. Use this to compute the (additive) cohomology of $\mathbb{C}P^3$. Can you work out the ring structure from this spectral sequence?

Hint: There are fibrations $S^1 \to S^{2n+1} \to \mathbb{C}P^n$.

Question 2.

Use the cohomological Serre spectral sequence associated to the path fibration

$$\Omega S^n \to PS^n \to S^n$$

to show the following: If n is odd, then

$$H^*(\Omega S^n) \cong \Gamma_{\mathbb{Z}}[x]$$

where $|x| = n - 1$. If n is even, then

$$H^*(\Omega S^n) \cong \Lambda_{\mathbb{Z}}[x] \otimes \Gamma_{\mathbb{Z}}[y]$$

where $|x| = n - 1$ and $|y| = 2n - 2$.

Here $\Lambda_{\mathbb{Z}}[x] \cong \mathbb{Z}[x]/(x^2)$ is the exterior algebra, while $\Lambda_{\mathbb{Z}}[x]$ is the divided polynomial algebra; the quotient of the free $\mathbb{Z}$-algebra $\mathbb{Z}\langle x_1, x_2, \ldots \rangle$ by the relations

$$x_n \cdot x_m = \binom{n+m}{n} x_{n+m}.$$

Note: This is not so easy! But give it a go, and if you don't figure it out, we will go over it in class.