

MA3408 Week 6

Drew Heard (drew.k.heard@ntnu.no)

February 16, 2021

Question 1.

Show, using the Serre spectral sequence, that if $S^k \rightarrow S^m \rightarrow S^n$ is a fibration with $n \geq 2$, then $k = n - 1$ and $m = 2n - 1$.

Question 2.

Show, using spectral sequences, the following result in homological algebra:

Given a commutative diagram

$$\begin{array}{ccccccccc} 0 & \longrightarrow & A & \longrightarrow & B & \longrightarrow & C & \longrightarrow & 0 \\ & & f \downarrow & & g \downarrow & & h \downarrow & & \\ 0 & \longrightarrow & A' & \longrightarrow & B' & \longrightarrow & C' & \longrightarrow & 0 \end{array}$$

in an abelian category with exact rows, there is a long exact sequence

$$0 \rightarrow \ker(f) \rightarrow \ker(g) \rightarrow \ker(h) \rightarrow \operatorname{coker}(f) \rightarrow \operatorname{coker}(g) \rightarrow \operatorname{coker}(h) \rightarrow 0.$$