MA3408 Week 5

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Question 1.

(i) Let X be an $(n-1)$-connected space, then $H^n(X;G) \cong \mathrm{Hom}_{\mathbb{Z}}(H_n(X), G)$.

(ii) Use this, and the theorem $[X, K(G,n)] \cong H^n(X;G)$ mentioned in class, to show that

$$[K(G,n), K(G',n)] = \mathrm{Hom}_{\mathbb{Z}}(G, G').$$

Question 2.

Show that a map between simply-connected CW complexes is a homotopy equivalence if its mapping cone is contractible.

Remark: Recall that the mapping cone of $f\colon X \to Y$ is the pushout

$$\begin{array}{ccc} X & \xrightarrow{f} & Y \\ \big\downarrow & & \big\downarrow \\ CX & \longrightarrow & Cf \end{array}$$

Equivalently, it is the quotient $Mf/j(X)$ of the mapping cylinder along the inclusion $j\colon X \to Mf$.

Question 3.

Show that if $S^k \to S^m \to S^n$ is a fiber bundle, then $k = n-1$ and $m = 2n-1$.

Question 4.

Show that a simply-connected closed 3-manifold is homotopy equivalent to S^3. (You may assume that every closed manifold is homotopy equivalent to a CW-complex.)

Remark: This is a little bit tricky, but you should try and compute the homology groups of such a manifold, and then use the homology Whitehead theorem.