

## MA3408 Week 4

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### Question 1.

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Let  $f: X \rightarrow Y$  be a map of topological spaces. Show that there is an induced map,  $\Gamma f: \Gamma X \rightarrow \Gamma Y$ , unique up to homotopy, between the CW-approximations to  $X$  and  $Y$ , such that the following diagram commutes:

$$\begin{array}{ccc} \Gamma X & \xrightarrow{\Gamma f} & \Gamma Y \\ \gamma \downarrow & & \downarrow \gamma \\ X & \xrightarrow{f} & Y \end{array} .$$

Deduce that CW-approximations are unique to homotopy.

### Question 2.

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Show that if  $X$  is  $n$ -connected, then  $\Sigma X$  is  $(n + 1)$ -connected.

### Question 3.

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Consider the equivalence relation  $\simeq_w$  generated by weak homotopy equivalence:  $X \simeq_w Y$  if there are spaces  $X = X_1, X_2, \dots, X_n = Y$  with weak homotopy equivalences  $X_i \rightarrow X_{i+1}$  or  $X_i \leftarrow X_{i+1}$  for each  $i$ . Show that  $X \simeq_w Y$  iff  $X$  and  $Y$  have a common CW approximation.