MA3408 Week 13

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Question 1.

Show that the inclusions $i_X \colon X \to X \vee Y$ and $i_Y \colon Y \to X \vee Y$ induce an isomorphism

$$i_X^* \oplus i_Y^* \colon \widetilde{K}^{-i}(X \vee Y) \to \widetilde{K}^{-i}(X) \oplus K^{-i}(Y)$$

for all $i \geq 0$.

Question 2.

Consider the following commutative diagram with exact rows and where all the f_n'' are isomorphisms:

$$\begin{tikzcd}
C''_{n+1} \arrow[r, "\delta_{n+1}"] \arrow[d, "f''_{n+1}"'] & C'_n \arrow[r, "i_n"] \arrow[d, "f'_n"] & C_n \arrow[r, "p_n"] \arrow[d, "f_n"] & C''_n \arrow[r, "\delta_n"] \arrow[d, "f''_n"] & C'_{n-1} \arrow[d, "f'_{n-1}"] \\
D''_{n+1} \arrow[r, "\delta'_{n+1}"'] & D'_n \arrow[r, "i'_n"'] & D_n \arrow[r, "p'_n"'] & D''_n \arrow[r, "\delta'_n"'] & D'_{n-1}
\end{tikzcd}$$

Show there is an exact sequence in which $\Delta_n = \delta_n \cdot (f_n'')^{-1} \cdot q_n \colon D_n \to C'_{n-1}$:

$$\cdots \to C'_n \xrightarrow{(i_n, f'_n)} C_n \oplus D'_n \xrightarrow{f_n - j_n} D_n \xrightarrow{\Delta_n} C'_{n-1} \to \cdots$$