MA3408 Week 13

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Question 1.

Let A be an abelian monoid. A group completion is an abelian group $K(A)$ together with a morphism of abelian monoids $i\colon A \to K(A)$, such that for any abelian group B and any map of abelian monoids $f\colon A \to B$, there exists a unique abelian group homomorphism $\widetilde{f}\colon K(A) \to B$ such that the diagram:

$$\begin{array}{ccc} A & \xrightarrow{i} & K(A) \\ & {\scriptstyle f}\searrow & \downarrow{\scriptstyle \exists!} \\ & & B \end{array}$$

commutes.

1. Give a construction of $K(A)$ for an abelian monoid $(A, \oplus)$.
2. Show that there is an adjunction
$$K\colon \text{AbMon} \leftrightarrows \text{AbGps}\colon U,$$
where U is the forgetful functor.
3. Show that if A is a commutative semi-ring (i.e., admits a multiplication which distriubtes over the sum), then $K(A)$ is in fact a commutative ring.
4. Consider the abelian monoid $\mathbb{Z}^+_\infty$ obtained from $\mathbb{Z}^+ = \{0, 1, 2, \ldots, \}$ under ordinary addition by adding the element ∞ and defining $k + \infty = \infty$ for every $k \in \mathbb{Z}^+_\infty$. Compute $K(\mathbb{Z}^+_\infty)$.

Question 2.

Show that $K(S^1) = \mathbb{Z}$.