

MA3408 Week 1

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1. Question 1 (Eckmann–Hilton argument)

Let M be a set, and let $*$ and \cdot be two binary operations on M , both with unit elements. Suppose that

$$(a * b) \cdot (c * d) = (a \cdot c) * (b \cdot d), \quad \forall a, b, c, d \in M.$$

Show that the units agree, the two operations agree, and the multiplication is commutative and associative.

Question 2.

Show that if G is a topological group with identity element e , then $\pi_1(G, e)$ is abelian.

Question 3.

If $\phi: X \rightarrow Y$ is a homotopy equivalence, then $\phi_*: \pi_n(X, x_0) \rightarrow \pi_n(Y, \phi(x_0))$ is a weak equivalence for all $n \geq 1$ and all $x_0 \in X$.

Note: This does not follow (immediately) from functionality, as f and its homotopy inverse need not preserve the basepoint.

Question 4.

Find spaces X, Y with $\pi_n X \cong \pi_n Y$ but $X \not\cong Y$.

Hint: What is the universal cover of $\mathbb{R}P^n$?