

# MA3408 Week 1

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## 1. Question 1 (Eckmann–Hilton argument)

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Let  $M$  be a set, and let  $*$  and  $\cdot$  be two binary operations on  $M$ , both with unit elements. Suppose that

$$(a * b) \cdot (c * d) = (a \cdot c) * (b \cdot d), \quad \forall a, b, c, d \in M.$$

Show that the units agree, the two operations agree, and the multiplication is commutative and associative.

## Question 2.

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Show that if  $G$  is a topological group with identity element  $e$ , then  $\pi_1(G, e)$  is abelian.

## Question 3.

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If  $\phi: X \rightarrow Y$  is a homotopy equivalence, then  $\phi_*: \pi_n(X, x_0) \rightarrow \pi_n(Y, \phi(x_0))$  is a weak equivalence for all  $n \geq 1$  and all  $x_0 \in X$ .

**Note:** This does not follow (immediately) from functionality, as  $f$  and its homotopy inverse need not preserve the basepoint.

## Question 4.

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Find spaces  $X, Y$  with  $\pi_n X \cong \pi_n Y$  but  $X \not\cong Y$ .

**Hint:** What is the universal cover of  $\mathbb{R}P^n$ ?