MA3402 HOMEWORK

Homeworks are intended as quick exercises to help solidify key ideas from the previous week. If you are spending a long time on them, stop, then talk it over with someone.

1. For Thursday 31 August 2023

(1) Let X be a smooth vector field on an open subset $U \subset \mathbb{R}^n$, f and g smooth functions on U. Show that $X(fg)$ satisfies the Leibniz rule. (Hint: start by considering each point $p \in U$ individually)

(2) Let $X = x\partial/\partial x + y\partial/\partial y$ be a vector field on $\mathbb{R}^3$.
Sketch X.
Let $f(x, y, z) = x^2 + y^2 + z^2$. Compute Xf.

(3) Let $f \in L_k(V)$. Show that f is alternating if and only if $f(v_1, \ldots, v_k) = 0$ whenever two of the vectors $v_1, \ldots, v_k$ are equal.

(4) Let $f \in A_2(V)$ and $g \in A_1(V)$. What is $(f \wedge g)(v_1, v_2, v_3)$? So why do we need $\frac{1}{k!l!}$ in the definition of the wedge product?