MA3402 SUGGESTED EXAM QUESTIONS

This is a compiled list of question suggestions kindly offered by MA3402 students.

(1) Let $F\colon N \to M$ be a C^∞ map of manifolds. For any $h \in C^\infty(M)$, show that $F^*(dh) = d(F^*h)$. (Prop 17.10)

(2) For $F\colon N \to M$ a smooth map of manifolds and $\omega \in \Omega^k(M)$, show that $dF^*\omega = F^*d\omega$. (Prop 19.5)

(3) For $F\colon N \to M$ a smooth map of manifolds and two forms ω, τ on M, show that $F^*(\omega \wedge \tau) = F^*(\omega) \wedge F^*(\tau)$.

(4) Suppose $(U, x^1, x^2, \dots, x^n)$ and $(V, y^1, y^2, \dots, y^n)$ are two coordinate charts on a manifold M. Then
$$\frac{\partial}{\partial x^j} = \sum_i \frac{\partial y^i}{\partial x^j}\frac{\partial}{\partial y^i}$$
on $U \cap V$. (i.e. Prove Proposition 8.10, or Prop 17.3, or maybe Prop 18.3)

(5) What is an atlas of a manifold? Give an atlas for S^1. What does it mean for an atlas to be oriented? Is your atlas for S^1 oriented?

(6) Let M be the square $\{(x, y) \in \mathbb{R}^2 \mid |x|, |y| \leq 1\}$ with the standard orientation from $\mathbb{R}^2$. Calculate the integral of $2x^2y^2dx \wedge dy$ over M both with and without Stokes' Theorem.

(7) For an n-manifold M
  (a) Calculate $H^0(M)$
  (b) Explain what the De Rham cohomology $H^k(M)$ is for $k > m$ and why.

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(8) Give an outline of the computation $H^1(S^1) \cong \mathbb{R}$. Then compute the cohomology of S^n for all $n > 0$.

(9) Given a chart $(U, x^1, \dots, x^n)$ containing a point p on a manifold M, write a basis for the tangent space T_pM. This is too short.

Would need to be part of a calculation or a proof.

(10) Explain the differences between a differential form, a differential of a function/form, a differential of a map (between manifolds), and a codifferential. Which one is the same as an exterior derivative?
OR Explain the differences between the differential of a map (between manifolds), the differential (of a function/differential form), and the differential (of a cochain complex). Which one is known as "the differential" in the context of commuting with the pullback? Good revision question, little short for the exam. Maybe connecting it to grad, curl, divergence or Maxwell's equations would make it more interesting.