MA3402 HOMEWORK

Homeworks are intended as quick exercises to help solidify key ideas from the previous week. If you are spending a long time on them, stop and talk it over with someone.

1. Homework 7

(1) Say a plane in $\mathbb{R}^3$ is *vertical* if it is defined by $ax + by = 0$ for some $(a, b) \neq (0, 0) \in \mathbb{R}^2$. Prove that $dx \wedge dy$ restricted to a vertical plane is 0. (Hint: what does $d(ax + by)$ tell us?)

(2) Let $\omega = dx^1 \wedge \cdots \wedge dx^n \in \Omega^n(\mathbb{R}^n)$ be the volume form on $\mathbb{R}^n$ and $X = \sum x^i \partial/\partial x^i$ be the radial vector field on $\mathbb{R}^n$. Compute the contraction $\iota_X \omega$.

(3) Prove that a pointwise orientation $[(X_1, \ldots, X_n)]$ on a manifold M is continuous if and only if every point $p \in M$ has a chart $(U, \phi) = (U, x^1, \ldots, x^n)$ around p such that

$$(\phi_* X_{1,q}, \ldots, \phi_* X_{n,q}) \sim \left(\left.\frac{\partial}{\partial r^1}\right|_{\phi(q)}, \ldots, \left.\frac{\partial}{\partial r^n}\right|_{\phi(q)} \right)$$

for every $q \in U$, i.e. the differential $\phi_* \colon T_q M \to T_{f(q)}\mathbb{R}^n$ carries the orientation of $T_q M$ to the standard orientation of $\mathbb{R}^n$.