

MA3402 HOMEWORK

Homeworks are intended as quick exercises to help solidify key ideas from the previous week. If you are spending a long time on them, stop and talk it over with someone.

1. FOR THURSDAY 21 SEPTEMBER 2023

- (1) Skim read Section 5 Manifolds, including the setup of the problems at the end in order to see examples of manifolds.
- (2) Check that a continuous map $f: M \rightarrow \mathbb{R}$ from a manifold to \mathbb{R} being smooth is independent of the choice of chart. i.e. Show that if $f \circ \phi^{-1}$ is smooth at $\phi(p)$ for a chart (U, ϕ) on M , then $f \circ \psi^{-1}$ is smooth at $\psi(p)$ for any other chart (V, ψ) with $p \in U \cap V$. (Hint: use transition maps).
- (3) Explicitly write down an atlas for S^1 and explain why it's an atlas to someone else (i.e. understand Example 5.16 from Tu).
- (4) Prove that $F: \mathbb{R} \rightarrow S^1$ given by $F(t) = (\cos t, \sin t)$ is smooth. (Hint: you will need an atlas for S^1)